STUDY OF OPTIMAL ORDERING POLICIES FOR TIME VARYING DECAY RATE OF INVENTORY UNDER DIFFERENT PAYMENT CONDITIONS

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THESIS

SUBMITTED TO GUJARAT UNIVERSITY, AHMEDABAD

FOR THE AWARD OF THE DEGREE OF

DOCTOR OF PHILOSOPHY

IN

MATHEMATICS

PREPARED UNDER THE GUIDANCE OF

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NOVEMBER 2003

CERTIFICATE

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ACKNOWLEDGEMENT

I wish to express my sincere thanks and profound gratitude to Dr. Nita H. Shah for her inspiring guidance and kind help in this venture of scientific research.

I am also thankful to Dr. N. R. Ladhawala (Head, Dept. of Mathematics, Gujarat University) for providing all the facilities for completing the work.

I earnestly thank the Principal Mr. L. M. Patel, my friends and colleagues Dr. Himanshu Pandya, Mr. Nisarg Bhatt, Mr. Ajit Bhavsar, Mr. Ashwin Prajapati and all the staff members of Smt. D. J. Shah Parivar Science College, Dholka, for their constant motivation, interest and co-operation.

How can I not remember the kindness and care shown to me by the family members of Dr. Nita H. Shah. I am also thankful to my research colleague Prof. Bhavin Shah for his constant motivation for completing the work.

I am extremely thankful to my dear student Mr. Rajendra Patel who has not seen day or night for helping me out in typesetting and the layout of the thesis. Without his help, the quality of the thesis would not have reached this height.

Last, but not the least, I am thankful to my mother, wife Dipa and all the members of my family for providing constant moral support and encouragement during the course of this work. My special thanks are due to my loving daughter Tosha who should have seen much more of her father during her holidays.

Ravi. M. Gor

RAVI GOR

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CHAPTER 1

INTRODUCTION

1.1 Introduction :

Operations Research is a scientific approach to problem solving for executive-decision making which requires the formulation of mathematical, economical and statistical models for decision and control problem to deal with situations arising out of risk and uncertainty. In the process of managing various subsystems of the organization, executives at different levels of the organization have to take several management decisions. These decisions are categorized into strategic decisions, tactical decisions and operational decisions. The strategic decisions are taken at the top management level where the definition of goals, policies and selection of decisions are in favor of the organizational objectives. The tactical decisions are taken at middle management level which include acquisition of resources, plant location, new products establishments and monitoring of budgets. The operational decisions, which are taken at the bottom level of management, include effective and efficient use of existing facilities and resources to carry out activities within budget constraints.

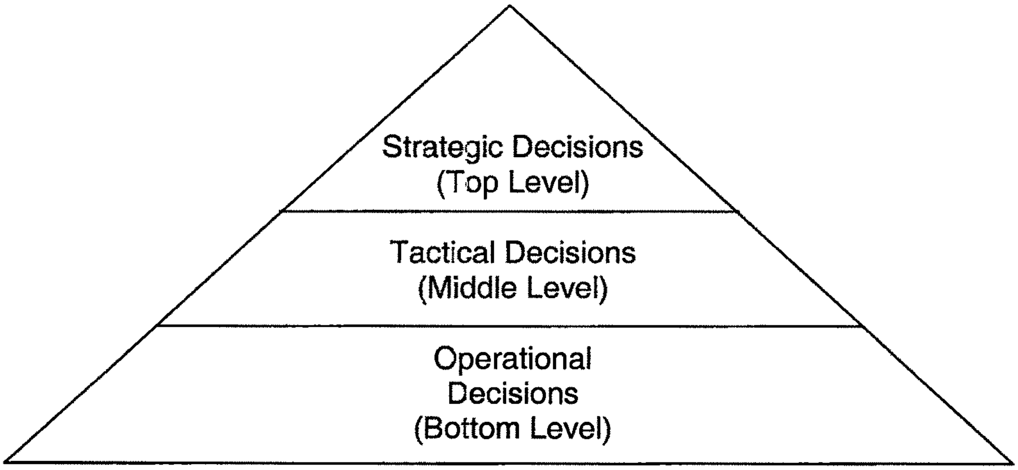


Fig. 1.1 Hierarchy of decisions at different levels of management

Thus, Operations Research may be considered as the application of scientific methods to decision making problems arising from operations involving integrated systems of people, machines and materials. The aim of the operations research is to find the best possible course of action of a decision making problem with or without some constraints. In departmental stores or shops, industrial unit, the stocking of items depends upon the various factors viz. demand, time of ordering, time gap between orders and actual receipts, deterioration, etc. So the problem for the managers / retailers is to have an equilibrium between over-stocking and under-stocking. The study of such type of problems is known as Inventory Control.

The term "inventory" is used to indicate raw materials in process, finished product, packaging, spares and others-stocked in order to meet a demand. Though inventory of materials is an idle resource- it is not meant for immediate use- it is almost essential to maintain some inventory for the smooth functioning of an enterprise. The main objective of inventory control is to reduce investment in inventories and ensuring that production process does not suffer at the same time.

The following two basic questions are to be answered to attain various objectives in an inventory control situation:

(1) What is the optimum quantity of an item that should be ordered?

(2) When should the order be placed?

In practice, it is not easy to determine a suitable inventory policy. An inventory problem is a problem of making optimal decisions regarding the above questions. Our aim is to develop the operating decisions that should be used to control the inventory system with the help of mathematical analysis. For this, we need to construct a mathematical model of the inventory system. Such a mathematical model is based on various assumptions and approximations.

1.2 Basic Concepts and Terminologies :

The inventory system depends on the following parameters:

- **Demand :** The number of units required per period is called ***demand***. It may be either deterministic or probabilistic. In the deterministic case, it is assumed that the quantities needed over subsequent periods of time are known with certainty. This may be expressed over equal periods of times in terms of known constant demands (referred as static demand) or in terms of known variable demands (referred as dynamic demand).

  Probabilistic demand occurs when requirements over a certain period of time are not known with certainty but their pattern can be described by a known probability distribution. The demand for a given period may be satisfied instantaneously at the beginning of the period or uniformly during that period. The effect of instantaneous and uniform demand reflects directly on the total cost of holding inventory.

- **Lead-Time** : The time gap between placing of an order and its actual arrival in the inventory is known as ***lead-time***.

  Lead-time has two components viz. administrative lead-time from initiation of procurement action until the placing of an order, and the delivery lead-time from placing of an order until the delivery of the ordered material.

- **Planning Horizon** : The time period over which the inventory level will be controlled is called the ***time horizon***. It may be finite or infinite depending upon the nature of the inventory system of the commodity.

- **Order Cycle** : The time period between placement of two successive orders is referred to as an ***order cycle***. The order may be placed on the basis of the following two types of inventory review systems.

  (A) **Continuous Review**: The record of the inventory level is checked continuously until reorder is reached when a new order is placed. This is also known as the ***two-bin system***. This divides the inventory into two parts and places it physically in two bins. Items are drawn from only one bin and when it is empty, a new order is placed. Demand is then satisfied from the second bin until the order is received. Upon receipt of the order, specified items are placed in the second bin and the remaining items are placed in the first bin. Every time this process is repeated.

  (B) **Periodic Review**: In this system the inventory levels are reviewed at equal time intervals and orders are placed at such intervals. The quantity ordered each time depends on the available inventory level at the time of review.

- **Reorder Level** : The level between maximum and minimum stock, at which purchasing activities must start for the replenishment is known as ***reorder level***.

- **Deterioration** : ***Decay*** or ***deterioration*** is defined as a physical process which hinders an item from being used for its original purpose such as (i) spoilage, as in perishable food stuffs, fruits and vegetables; (ii) physical depletion, as in pilferage or evaporation of volatile liquids such as gasoline, perfumes, alcohol; (iii) decay as in radioactive substances, degradation, as in electronic components or loss of potency as in photographic films, pharmaceutical drugs, fertilizers etc. For perishable goods such as dairy products, bakery items, vegetables, etc. it is observed that the age of inventory has a negative impact on consumer confidence for reasons such as (a) proximity to expiry dates (for applicable items) , (b) detrimental effects on the quality if the product because of aging of inventory, and (c) general conception that an item lying unsold for a long time may be of inferior quality.

The rate of deterioration is measured by the fraction deteriorated per quantity per time unit and it may be constant or vary with time or stocked units.

- **Ordering Cost** : ***Ordering cost*** is the cost associated with ordering of raw material for the production purposes. It includes advertisement cost, consumption of stationary and postage, telephone charges, telegrams, rents for space used by the purchasing department.
- **Purchase Cost** : The cost of purchasing a unit of an item is known as ***purchase cost***. The purchase price plays an important role when quantity discounts are considered for purchases above a certain quantity or when economies of scale suggest that the per unit purchase cost can be reduced by a larger production run.
- **Carrying (Holding) Cost** : The ***carrying cost*** is associated with carrying inventory. This cost includes the costs such as rent for warehouse used for storage, interest on the money locked-up, insurance of stored equipment, production, taxes, depreciation of equipment and furniture used, etc.
- **Shortage Cost** : The penalty cost for running out of stock i.e. when an item cannot be supplied on the customer's demand is known as ***shortage cost***. This cost includes the loss of potential profit through sales of items and loss of good will, in terms of permanent loss of customers and its associated lost profit in future sales.

- **Selling Price** : When the demand for certain commodity is affected by the quantity stocked, decision problem has profit maximization as an objective, includes the revenue from selling.

1.3 Literature Survey:

Harris, in 1915, first developed the analysis of an inventory system and derived the classical lot size formula, though Erlenkotter (1989) reported that the earliest model was developed by Harris in 1913. After a few years, Wilson (1934) developed independently the same formula obtained by Harris. This formula has been named as Harris-Wilson formula or Wilson's formula which was formulated under certain simple assumptions, such as known and uniform demand, without shortages, infinite replenishments, negligible lead time, etc. However, in real-life situations, inventory loss may be due to deterioration. Then problem for decision makers is how to control and maintain inventories of deteriorating items.

During the last two decades, researchers are engaged in analyzing inventory models for deteriorating items such as volatile liquids, blood, medicines, electronic components, fashion goods, fruits and vegetables etc. Whitin (1957) studied deterioration at the end of the storage period; for example, for the fashion goods industry. Berrotoni (1962) found that both leakage failure of dry batteries and life expectancy of ethical drugs could be expressible in terms of Weibull distributions, in discussing the problem of fitting empirical data to mathematical distributions. In both cases, the rate of deterioration increased with age or the longer the items remained unused, the higher the rate at which they fail. At some point all units that have not been used would have failed while in inventory. Emmons (1968) considered the decay of radioactive nuclide generators. Here the decay is the total usage. Ghare and Schrader

(1963) first developed a mathematical model with a constant decaying rate. They classified the phenomena of inventory deterioration into three types, viz direct spoilage, physical depletion and deterioration.

The Weibull density function is

$$F(t) = \alpha\beta t^{\beta-1} e^{-\alpha t^{\beta}}$$

where

α denotes the scale parameter; $\alpha > 0$

β denotes the shape parameter; $\beta > 0$

t denotes the time ; $t > 0$

When used in the context of economic order quantities, the Weibull distribution will provide a probability density function that gives the time to deterioration.

Using this concept the Ghare and Schrader's model was extended by Covert and Philip. Covert and Philip (1973) discussed an EOQ model for items with Weibull distribution deterioration.

The deterioration rate given by the Weibull distribution is

$$\theta(t) = \alpha\,\beta\,t^{\beta-1} \qquad ,0 \le t \le T \qquad (1.3.1)$$

where

α - scale parameter, $0 \le \alpha < 1$,

β - shape parameter, $\beta \ge 1$

t - time to deterioration, $t > 0$

The scale parameter α of the weibull distribution has same units as that of time t. Hence, a change in the scale parameter α has the same effect on the distribution as a change of the time scale. Keeping β, the shape parameter constant, if α is increased, the distribution gets stretched out to the right and its height decreases while maintaining its shape and location. If α is decreased, the distribution gets pushed in towards zero and its height increases.

The shape parameter β defines the slope of the weibull distribution. The probability density function represents on – hand inventory deterioration that may have an increasing, decreasing or constant rate depending on the value of β (see Walpole and Myers (1978)) and the practical applications of the weibull distribution is described in Berrottoni (1962).

When $\beta > 1$, deteriorating rate increases with time;

e.g. fish, vegetables, medicines.

When $\beta < 1$, deteriorating rate decreases with time;

e.g. light bulb where the initial breakdown rate may be higher due to irregular voltage and handling.

When $\beta = 1$, deteriorating rate is constant;

e.g. electronic products.

Here the two parameter weibull distribution is reduced to an exponential distribution.

Since then, lot of work has been done by Misra (1975), Shah (1977), Dave and Patel (1981), Hollier and Mak (1983), Heng et. al (1991), Hariga (1995), Wee (1995) on deteriorating inventory systems. The review articles by Raafat (1991), Shah and Shah (2000), Goyal & Giri (2001) gives a complete and up–to–day survey of published inventory literature for the deteriorating inventory models. Most of the addressed articles consider the effect of constant deterioration, which is a function of the on hand level of inventory.

On the other hand, in the developed mathematical models, it is assumed that, the shortages are either completely backlogged or completely lost. Shah and Jaiswal (1977) and Aggarwal (1978) presented and reformulated an order-level inventory model with a constant rate of deterioration. Researchers derived models under the assumption that a fraction of the demand will be lost while the remaining fraction is backlogged. (See Wee (1995), and Yan and Cheng (1998)).

Second stringent assumption in traditional inventory economic order quantity model was that the purchaser must pay for the items as soon as it is received by the system. In the classical EOQ model, it was tacitly assumed that the supplier is paid for the items immediately on the receipt of the goods. However, in practice, the supplier may provide a cash discount and / or a credit period to the customer for the settlement of the amount within the permitted fixed settlement period. Thus, the delay in payment to the supplier is a kind of price discount, since paying later indirectly reduces the purchase cost and it can encourage customers to increase their order quantity.

Goyal (1985) derived an EOQ model under the conditions of permissible delay in payments. Shah (1993) extended Goyal's model, for deteriorating items. Independently, Jaggi and Aggarwal (1995) established an inventory model for deteriorating items when delay in payments was permissible. Jamal et al. (1997) then further generalized the model to allow for shortages. Shah (1993) developed a probabilistic version of Goyal's model. Shah (1993) extended it for deteriorating items. Shah and Shah (1998) developed an EOQ model for constant rate of deteriorating items in which time is treated as discrete variable, deterioration of units as continuous variable and demand as a random variable. Shah (1997) developed a probabilistic order level system with lead time when delay in payments is permissible. Shah and Shah (1996) derived optimal ordering policies under the conditions of extended payment privileges. Jaggi and Aggarwal (1994) presented the economic ordering policies for deteriorating items in the presence of trade credit using a discounted cash-flows (DCF) approach which permits a proper recognition of the financial implication of the opportunity cost and out-flow costs in inventory analysis. Shah and Shah (1996) derived optimal ordering policies under conditions of extended payment privileges. Jamal, Sarkar and Wang (2000) formulated model when retailer can pay the wholesaler either at the end of the credit period or later incurring interest charges on the unpaid balance for the overdue period. They developed retailer's policy for optimal cycle and payment times for a retailer in a deteriorating-item inventory scenario where a wholesaler allows a specified credit period to the retailer for payment without penalty. Hwang and Shinn (1997) developed the model for determining the retailer's optimal price and lot-size simultaneously when the supplier permits delay in payments for an order of a product whose demand rate is a function of constant price elasticity. Other articles related to this topic are Shinn et al (1996), Chung (1998), Chu et al. (1998) and Teng (2002).

Recently, Arcelus et al (2001) compared policies of price discount and trade credit. They studied comparison of retailer's response to special sales in two strategies of price discount and trade credit. Arcelus et al (2003) developed mathematical model for retailer's maximum profit when supplier offers credit period and / or price discount on the purchase of regular order when units in inventory are subject to constant rate of deterioration. In this paper, an attempt is made to derive an EOQ model for time dependent deteriorating items in which the supplier provides not only a cash discount but also a fixed credit period to the customer. For example, the supplier offers a 2% discount off the price if the payment is made within 10 days, otherwise the full payment of the order is due within 30 days. This credit is denoted as "2/10 , net 30" (e.g. See Brigham (1995)).

The interesting papers related to trade credits are of Davis and Gaither (1985), Arcelus and Srinivasan (1993,1995,2001), Shah (1993,1997), Khouja and Mehrez (1996), Hwang and Shinn (1997), Chu et al (1998), Chung (1998), Teng (2002).

The above models were developed with the assumption that inflation does not have significant role to play on the inventory policy. However from a financial point of view, an inventory represents a capital investment and must compete with other asset's for a firm's limited capital funds. Thus, it is important to consider the effects of inflation on the inventory system. Buzacott (1975), Misra (1975) and Bierman and Thomas (1977) analyzed inventory decisions under an inflationary condition for the EOQ model. Misra (1975) developed mathematical model by considering the time value of money and different inflation rates. Brahmbhatt (1982) developed an EOQ model under a variable inflation rate and mark-up prices. Chandra and Bahner (1985) derived model to discuss the effects of inflation and time value of money on optimal order policies. Datta and Pal (1991) developed a model with linear time dependent demand rates and shortages to study the effects of inflation and time value of money on a finite planning horizon. Liao et al (2000) presented a model for deteriorating items under inflation when delay in payments is permissible.

Stock dependent demand models are the models where demand rate is proportional to the inventory level. In case of deterministic models, it is optional to let the inventory level to be zero, but not so in stock-dependent demand rate models. As the inventory level decreases, there are lost sales. In this type of demand pattern, the quantity demand decreases as the inventory level decreases, resulting in lower sales and lower profits. To keep sales higher, the inventory level should be higher which in turn results in the higher holding cost and procurement cost.

It has been observed in supermarkets that the demand rate is usually influenced by the amount of stock level. Levin et al (1972) quoted "at times, the presence of inventory has motivational effect on people around it. It is common belief that large piles of goods displayed in a supermarket will attract the customers to buy more". Silver and Peterson (1982) noted that sales at the retail level tend to be proportional to the amount of inventory displayed. This fact attracted a number of researchers to develop EOQ models focused on stock-dependent demand rate patterns. Gupta and Vrat (1986) considered demand rate to be a function of initial stock level. Mandal and Phaujdar (1989) formulated production inventory model for deteriorating items with uniform rate of production and linearly stock-dependent demand. Dixit and Shah (2001) extended the above model for more general demand. Baker and Urban (1988) developed model in which sales were directly affected by the allocation of shelf-space. Datta and Pal (1990) extended Mandal and Phaujdar's (1989) model for deteriorating items with the assumption that the demand rate is linear function of the on-hand inventory allowing shortages completely backlogged, for both finite and infinite time-horizons. Some of the recent work in this area is by Padmanabham and Vrat (1995), Ray and Chaudhuri (1997), Sarker et al (1997), Giri and Chaudhuri (1998), Mandal and Maiti (1999), Chung (2002).

Shah and Jaiswal (1977) and Aggarwal (1978) derived an order level inventory model for deteriorating items with time-proportional demand when shortages were not allowed. Dave and Patel (1981) developed an inventory model for deteriorating items with time proportional demand. Sachan (1984) extended above model by allowing shortages. Hariga (1996) generalized the demand pattern to any log-concave function. Teng et al. (1999) and Yang et al. (2001) further generalized the demand function to include any non-negative, continuous function that fluctuates with time. Liao et al (2000) developed an inventory model for stock-dependent consumption rate when delay in payments is permissible.

1.4 Outline of the thesis

The proposed thesis has been divided into five chapters on the basis of the structure of the different models, i.e.

- **Chapter-1:** Introduction
- **Chapter-2:** Inventory models with time dependent deterioration of units
- **Chapter-3:** Inventory models with time dependent deterioration of units under conditions of permissible delay in payments
- **Chapter-4:** An EOQ model for deteriorating items with price dependent demand under supplier credits
- **Chapter-5:** An EOQ model for deteriorating items with selling price and stock dependent demand during inflation under supplier credits

The following is a chapter wise description of inventory models dealt with in the proposed thesis.

Chapter 1 contains an introduction giving an overview of the development on inventory systems under different assumptions.

Chapter 2 discusses inventory models with time dependent deterioration of units

In this chapter, two inventory models have been proposed and formulated. **Section-2.1** deals with a lot-size model for items with time dependent deterioration. Here, a lot size inventory model is developed for deteriorating items with a time dependent rate of deterioration which follows Weibull density function. The EOQ formula is derived under the assumptions of constant demand, zero lead time and no shortages. The analytic proofs are given to support each assertion of parameter dependence. A numerical example is given to show the solution pattern. **Section-2.2** is a mathematical development of an order level lot-size model with time dependent deterioration. This model is an extension of the model given in section 2.1. It is developed under the assumptions of constant demand, zero lead time and by allowing shortages. Again, the Weibull density function is used to represent the time dependent deterioration. Shortages are allowed and are completely backlogged. Analytic proof of parameter dependence is given. A hypothetical numerical example is used to support the solution procedure.

Chapter 3 is Inventory models with time dependent deterioration of units under conditions of permissible delay in payments. In this chapter, three inventory models have been proposed. The mathematical models are developed to determine the optimal ordering policy for variable deterioration of units in an inventory system under permissible delay in payments. The following two scenarios are discussed in Section 3.1 and 3.2 :

Scenario-1: When permissible delay period in payments is less than the cycle time

Scenario-2: When permissible delay period in payments is greater than the cycle time

Section-3.1 deals with a lot-size model with variable deterioration rate under supplier credits. The model is developed under assumption of instantaneous and infinite replenishments and no shortages. The deterioration of items in the inventory follows the Weibull density function. The Newton-Raphson method has been used to find the optimum solutions. An easy-to-use algorithm to find the solution is given. Sensitivity analysis of the optimal solution with respect to the parameters of the system is carried out.

Section-3.2 is an order-level lot size model with time dependent deterioration and permissible delay in payments. Shortages are very important especially in a model that considers permissible delay in payments due to the fact that shortages can affect the quantity to be replenished to benefit from the permissible delay period. This model is an extension of the previous model. Shortages are allowed here. The rest of the assumptions remain the same as that of the model in section 3.1. The mathematical methodology to find the solution of the equations is explained in detail and a special case is also discussed. Sensitivity analysis is carried out at the end and a few directions for future research are discussed.

Section-3.3 is an EOQ model for deteriorating items with two parameter Weibull distribution deterioration under supplier credits. In this chapter, an inventory model is developed for deteriorating items with two parameter Weibull distribution, in which the supplier provides both cash discount and credit period to the customer. Also, in the model, lead time is zero, replenishment is infinite and shortages are not allowed.

In the model the following four cases are discussed based on the provisions of discount and permissible delay periods.

Case 1. The payment is paid at M_1 to get a cash discount and cycle time $T \geq M_1$

Case 2. The customer pays in full at M_1 to get a cash discount but cycle time $T<M_1$

Case 3. The payment is paid at time M to the permissible credit and cycle time $T \geq M$

Case 4. The customer pays in full at M and cycle time $T<M$

The Taylor's series approximation is used to determine the mathematical results. A numerical example is provided to verify the results obtained in the market. A few directions for future research are also given.

Chapter 4 is An EOQ model for deteriorating items with price dependent demand under supplier credits. In this chapter, an EOQ model for constant rate of deterioration of units and selling price dependent demand, in which the supplier provides not only a cash discount but also provides a credit period to the customer, is discussed.

The demand is taken as

$R(p) = a - bp \quad (a > 0, b > 0, \quad a >> b)$ where p denotes the selling price of the item.

In this model the following four cases are discussed based on the provisions of discount and permissible delay periods.

Case 1. The payment is paid at M_1 to get a cash discount and cycle time $T \geq M_1$

Case 2. The customer pays in full at M_1 to get a cash discount but cycle time $T<M_1$

Case 3. The payment is paid at time M to the permissible credit and cycle time $T \geq M$

Case 4. The customer pays in full at M and cycle time $T<M$

Also, in the model, lead time is zero, replenishment is infinite and shortages are not allowed. The necessary and sufficient conditions for finding the optimal solution to the problem are derived. Taylor's series approximation is used to obtain the solution. The effects of changes in parametric values are studied on the decision variables and the objective function.

Chapter 5 is an EOQ model for deteriorating items with selling price and stock dependent demand under conditions of permissible delay in payments. In this chapter, an inventory model is developed for deteriorating items for which the demand is dependent on the selling price as well as the stock. The units in inventory are subject to constant rate of deterioration. Shortages are not allowed and the supplier provides a cash discount and a permissible delay in payments.

Demand is taken as $R(p, I(t)) = \alpha - \beta p + \delta I(t)$,

Where α, is the fixed demand and $\alpha > 0$

β and δ constants, $\alpha >> \beta$, $\alpha >> \delta$

The optimal solution is characterized to optimize the net profit. An easy-to-use algorithm is given to find the optimal selling price, the optimal order quantity and replenishment cycle time that maximizes the net profit. At the end, a numerical example is given to illustrate the theoretical results and sensitivity analysis of parameters on the optimal solutions is carried out.

The direction for future research plans follows chapter 5.

List of papers published / presented / accepted follows the future research plans.

The thesis concludes with Bibliography.

The Assumptions used throughout the thesis

ASSUMPTIONS – A.1

The following are the assumptions used in the thesis.

1. The inventory system deals with only one item.
2. The replenishment rate is infinite.
3. Lead-time is zero.
4. The deterioration rate of the units in the inventory follows the Weibull density function given by equation 1.3.1.
5. There is no repair or replacement of deteriorated units during the period under consideration.
6. During the time the account is not settled, generated sales revenue is deposited in an interest bearing account. At the end of the trade credit period, the customer pays off all the units sold, keeps profits and starts paying for the interest charges on the items in stock.

Additional assumptions, if any, will be mentioned in the relevant chapters.

The Notations used throughout the thesis

NOTATIONS – N.1

The following are the notations used in the thesis

A	:	Ordering cost of inventory, $/ per order
C	:	The purchase cost per unit
P	:	The selling price per unit
T	:	Optimum cycle time when Q units are depleted to zero due to both demand and deterioration
$I(t)$	:	The inventory level at time t, $0 \le t \le T$
Q	:	The optimum order quantity
I	:	The inventory carrying charge fraction / unit / annum excluding interest charges paid
I_c	:	Interest charged per $ in stock per annum by the supplier
I_e	:	Interest earned per $ per annum
h	:	The inventory holding cost per unit per time unit excluding interest charges
π	:	the shortage cost/unit
SC	:	shortage cost/cycle
CD	:	cost of deterioration/cycle
IHC	:	inventory holding cost/cycle
R	:	The demand rate (units per time unit)
IE	:	interest earned per cycle
IC	:	interest charged per cycle
$D(T)$	:	number of units deteriorated during a cycle time T

Additional notations, if any, will be mentioned in the relevant chapters.

CHAPTER 2

INVENTORY MODELS WITH TIME DEPENDENT DETERIORATION OF UNITS

2.0 Introduction:

In this chapter, two inventory models have been formulated.

They are

Section 2.1 - A lot size model for deteriorating items with time dependent deterioration.

Section 2.2 - An order level lot size model for time dependent deterioration.

2.1 A LOT SIZE MODEL FOR DETERIORATING ITEMS WITH TIME DEPENDENT DETERIORATION

In this section a lot-size inventory model is developed for deteriorating items with a time dependent rate of deterioration. The EOQ formula is derived under assumptions of constant demand, zero lead-time and no shortages. It is shown that the results can be reduced to known models. Analytic proof of parameter dependence is given. A numerical example is used to show the solution pattern.

2.1.1 Assumptions and Notations :

The lot-size inventory model for deteriorating items will be developed using the following additional assumptions other than those given in A.1:

1. Shortages are not allowed.
2. The distribution of the time to deterioration of the item is as given in Equation 1.3.1
3. Q is a decision variable.

2.1.2 Mathematical Formulation :

Let *Q(t)*, $0 \le t \le T$ denotes on-hand inventory of units at time *t*. The instantaneous state of *Q(t)* for any instant of time , follows the differential equation

$$\frac{dQ(t)}{dt} + \theta(t)Q(t) = -R \qquad 0 \le t \le T, \qquad (2.1.2.1)$$

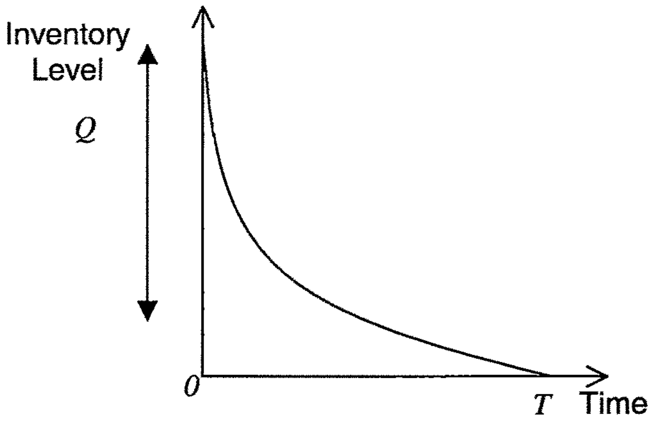


Fig. 2.1.2.1

with initial condition $Q(0) = Q$ and boundary condition $Q(T) = 0$.

Equation (2.1.2.1) is the first order linear differential equation whose general solution is

$$Q(t) = -\mathrm{R}e^{-\alpha t^{\beta}} \int_{0}^{t} \alpha t^{\beta} dt + k e^{-\alpha t^{\beta}}$$

with boundary condition $Q(T)= 0$,we get particular solution as

$$Q(t) = \mathrm{R}e^{-\alpha t^{\beta}} \int_{t}^{T} e^{-\alpha t^{\beta}} dt \qquad (2.1.2.2)$$

$$Q(t) = \mathrm{R}e^{-\alpha t^{\beta}} \sum_{n=0}^{\infty} \frac{\alpha^{n}}{n!(n\beta+1)} (T^{n\beta+1} - t^{n\beta+1})$$

and hence $Q(0) = Q$ gives

$$Q = R \sum_{n=0}^{\infty} \frac{\alpha^{n} T^{n\beta+1}}{n!(n\beta+1)} \qquad (2.1.2.3)$$

Number of units that deteriorate during $[0, T]$ is given by

$$D(T) = Q - RT$$

$$= R \left[\sum_{n=0}^{\infty} \frac{\alpha^{n} T^{n\beta+1}}{n!(n\beta+1)} - T \right]$$

Hence cost due to deterioration per time unit is

$$CD = \frac{CD(T)}{T}$$

$$= \frac{CR}{T}\left[\sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)} - T\right] \tag{2.1.2.4}$$

The Ordering cost per time unit is $OC = \frac{A}{T}$ (2.1.2.5)

Now, inventory on-hand per time unit $I(T) = \frac{1}{T}\int_0^T Q(t)dt$

$$= \frac{R}{T}\int_0^T e^{-\alpha t^\beta} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!(n\beta+1)}(T^{n\beta+1} - t^{n\beta+1})dt$$

$$= \frac{R}{T}\sum_{n=0}^{\infty} \frac{\alpha^n}{n!(n\beta+1)}\int_0^T e^{-\alpha t^\beta}(T^{n\beta+1} - t^{n\beta+1})dt$$

$$= \frac{R}{2T}\sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta+1)^2} T^{2(n\beta+1)}$$

Hence, inventory holding cost, *IHC* per time unit is = *CI I(T)*

$$= \frac{CIR}{2T}\sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta+1)^2} T^{2(n\beta+1)} \tag{2.1.2.6}$$

Using Equations (2.1.2.4) - (2.1.2.6), we get total cost, *K(T)* of an inventory system per time unit as

$$K(T) = CD + OC + IHC$$

$$= CR\left[\sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta}}{n!(n\beta+1)} - 1\right] + \frac{A}{T} + \frac{CIR}{2}\sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta+1)^2} T^{2n\beta+1}$$

The optimum value of cycle time $T = T_0$ can be obtained by solving $\frac{dK(T)}{dT} = 0$

i.e. $$CR \sum_{n=0}^{\infty} \frac{n\beta\alpha^n T^{n\beta+1}}{n!(n\beta+1)} - A + \frac{CIR}{2} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}(2n\beta+1)}{(n!)^2(n\beta+1)^2} T^{2(n\beta+1)} = 0$$

using suitable numerical method. The above equation can be solved for any sets of parametric values. The series can be truncated if αt^{β} is less than one because $0< \alpha < 1$ while T is time, the proper selection of the dimensions of time will allow the convergence of the solution

2.1.3 Special Cases :

1) Ghare and Schrader's (1963) model can be obtained by putting $\alpha = \theta$ and $\beta = 1$ in the above model.

$$Q = R \sum_{n=0}^{\infty} \frac{\theta^n T^{n+1}}{n!(n+1)}$$

$$K(T) = CR\{ \sum_{n=0}^{\infty} \frac{\theta^n T^n}{n!(n+1)} - 1\} + \frac{A}{T} + \frac{CIR}{2} \sum_{n=0}^{\infty} \frac{(-1)^n \theta^{2n} T^{2n+1}}{(n!)^2(n+1)^2}$$

2) By taking $\alpha = 0$, the model reduces to that of Naddor (1966)

$$Q = RT$$

$$K(T) = \frac{A}{T} + \frac{CIRT}{2}$$

2.1.4 Assertions :

1. Total cost of an inventory system per time unit increases with respect to the scale parameter α

   Proof :

$$\frac{dK}{d\alpha} = CR\sum_{n=0}^{\infty}\frac{n\alpha^{n-1}T^{n\beta}}{n!(n\beta+1)} + \frac{CIR}{2}\sum_{n=1}^{\infty}\frac{(-1)^n 2n\alpha^{2n-1}}{(n!)^2(n\beta+1)^2}T^{2n\beta+1} \quad > 0 \quad \forall\ T.$$

2. Total cost of an inventory system per time unit increases with respect to the ordering cost per order.

   Proof :

$$\frac{dK}{dA} = \frac{1}{T} \quad > 0 \quad \forall\ T$$

3. Increase in demand increases total cost of an inventory system per time unit.

   Proof :

$$\frac{dK}{dR} = C\left[\sum_{n=0}^{\infty}\frac{\alpha^n T^{n\beta}}{n!(n\beta+1)} - 1\right] + \frac{CI}{2}\sum_{n=0}^{\infty}\frac{(-1)^n\alpha^{2n}}{(n!)^2(n\beta+1)^2}T^{2n\beta+1} \quad > 0 \quad \forall\ T$$

4. Under the assumption that $0 \le \alpha \le 1$, neglecting α^2 and its higher powers, increase in the shape parameter increases the total cost of an inventory system per time unit.

   Proof :

$$\frac{dK}{dR} = \frac{R\alpha T^{\beta-1}}{(\beta+1)(\beta+2)}\left[\frac{2CIT^2}{\beta+2} + \frac{CI\beta T + C(\beta+2)(\beta^2+\beta-T)}{\beta+1}\right] \quad > 0 \quad \forall\ T$$

2.1.5 Numerical Example and observations :

A hypothetical model is developed using the following parametric values.

$$[C, I, R, A, \alpha, \beta] = [20, 12\%, 1000, 150, 0.02, 1.5]$$

In the following tables, the effect of various parameters on optimum cycle time, optimal procurement quantity and minimum cost of an inventory system is studied.

Table 2.1.5.1 Variations in α

$[C, I, R, A, \beta] = [20, 12\%, 1000, 150, 1.5]$

α	0.02	0.03	0.04	0.05	0.06
T	0.3342	0.3260	0.3185	0.3116	0.3053
Q	334.75	326.74	319.45	312.78	306.63
K	880.78	895.99	910.67	924.88	938.65

Table 2.1.5.2 Variations in A

$[C, I, R, \alpha, \beta] = [20, 12\%, 1000, 0.02, 1.5]$

A	150	200	250	300	350
T	0.3342	0.3843	0.4283	0.4678	0.5040
Q	334.75	385.10	429.26	469.02	505.46
K	880.78	1019.70	1142.51	1253.55	1356.49

Table 2.1.5.3 Variations in R

$[C, I, A, \alpha, \beta] = [20, 12\%, 150, 0.02, 1.5]$

R	1200	1400	1600	1800	2000
T	0.3058	0.2837	0.2658	0.2509	0.2384
Q	367.52	397.69	425.81	452.23	477.25
K	963.33	1039.19	1109.75	1175.98	1238.60

Table 2.1.5.4 Variations in β

$[C, I, A, R., \alpha] = [20, 12\%, 150, 1000, 0.02]$

β	1.2	1.3	1.4	1.5	1.6
T	0.3297	0.3312	0.3327	0.3342	0.3356
Q	330.53	331.96	333.37	334.75	336.08
K	898.91	891.68	885.80	880.78	876.49

Table 2.1.5.5 Variations in C

$[I, A, R, \alpha, \beta] = [12\%, 150, 1000, 0.02, 1.5]$

C	20	25	30	35	40
T	0.3342	0.2985	0.2743	0.2544	0.2384
Q	334.75	300.22	274.65	254.72	238.62
K	880.78	982.86	1075.06	1159.79	1238.60

Table 2.1.5.6 Variations in I

$[C, A, R., \alpha, \beta] = [20, 150, 1000, 0.02, 1.5]$

I	12%	13%	14%	15%	16%
T	0.3342	0.3227	0.3122	0.3027	0.2940
Q	334.75	323.17	312.70	303.17	294.44
K	880.78	913.66	945.45	976.23	1006.11

Observations :

Table No.	Observations
2.1.5.1	Increase in deterioration rate (α) decreases cycle time and increases total cost of the inventory system
2.1.5.2	Increase in the ordering cost (A) increases the cycle time and the total cost of an inventory system significantly.
2.1.5.3	Increase in demand rate (R) decreases the cycle time and increases the procurement quantity and the total cost of an inventory system significantly.
2.1.5.4	Increase in the shape parameter (β) increases the cycle time and decreases the total cost of an inventory system.
2.1.5.5	Increase in the purchase cost (C) decreases cycle time and procurement quantity while increases the total cost of an inventory system significantly.

2.1.5.6 Increase in the carrying charge fraction (I) per annum reduces the cycle time and the procurement quantity and increases the total cost of an inventory system.

2.2. AN ORDER LEVEL LOT-SIZE MODEL WITH TIME DEPENDENT DETERIORATION

In this section, a mathematical model is developed with same assumptions as those of section 2.1 by allowing shortages which are completely backlogged.

2.2.1 Assumptions and Notations :

The order-level lot-size inventory model for time dependent deterioration of units is developed under following additional assumptions and notations other than those given in A.1 and N.1 earlier.

1. Shortages are allowed and completely back-logged.
2. The shortage cost, π, per unit is constant.
3. The distribution of the time for deterioration of the item is as given in 1.3.1

2.2.2 Mathematical Formulation :

Suppose that the system carries inventory during $(0, T_1)$ and runs with shortages during (T_1,T) (fig. 2.2.2.1). The instantaneous state of $Q(t)$ which denotes on-hand inventory of units at time t, follows the differential equation.

$$\frac{dQ(t)}{dt}+\theta(t)Q(t)=-R, \qquad 0\le t\le T_1 \qquad (2.2.2.1)$$

$$\frac{dQ(t)}{dt}=-R, \qquad T_1\le t\le T$$

with initial condition $Q(T_1) = 0$ and $Q(0) = Q$

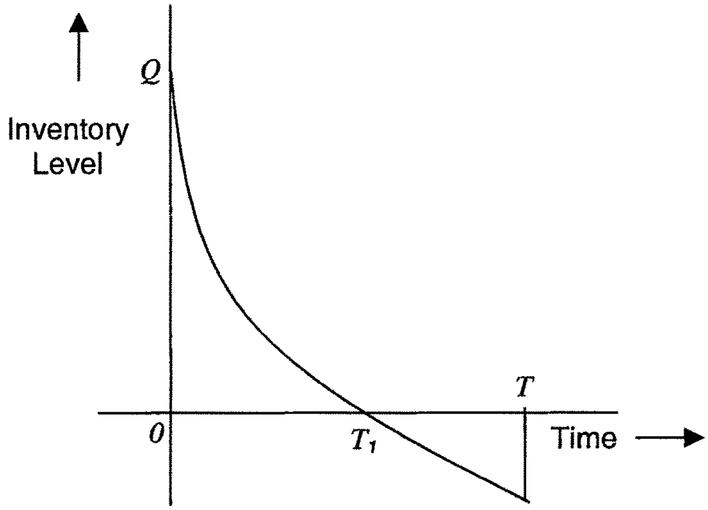


Fig. 2.2.2.1

Equation (2.2.2.1) is the first order linear differential equation whose solution using boundary condition $Q(T_1) = 0$ is given by

$$Q(t) = \text{R}e^{-\alpha t^{\beta}} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!(n\beta+1)} (T_1^{n\beta+1} - t^{n\beta+1}) \quad (2.2.2.2)$$

and hence

$$Q = Q(0) = R \sum_{n=0}^{\infty} \frac{\alpha^n T_1^{n\beta+1}}{n!(n\beta+1)} \quad (2.2.2.3)$$

Number of units that deteriorate during $[0, T_1]$ is given by

$$D(T) = Q - RT$$

$$= R[\sum_{n=0}^{\infty} \frac{\alpha^n T_1^{n\beta+1}}{n!(n\beta+1)} - T]$$

Hence cost due to deterioration per time unit is

$$CD = \frac{CD(T)}{T} = \frac{CR}{T} [\sum_{n=0}^{\infty} \frac{\alpha^n T_1^{n\beta+1}}{n!(n\beta+1)} - T] \quad (2.2.2.4)$$

The ordering cost per time unit is

$$OC = \frac{A}{T} \tag{2.2.2.5}$$

The inventory on-hand per time unit is given by

$$I(T) = \frac{1}{T}\int_{0}^{T_1} Q(t)dt$$

$$= \frac{R}{2T}\sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta+1)^2} T_1^{2(n\beta+1)}$$

Hence, inventory holding cost, *IHC*, per time unit is

$$IHC = \frac{hR}{2T}\sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta+1)^2} T_1^{2(n\beta+1)} \tag{2.2.2.6}$$

The shortage cost, *SC*, per time unit is

$$SC = \frac{\pi R}{T}\int_{T_1}^{T} t\,dt$$

$$= \frac{\pi R}{2T}(T - T_1)^2 \tag{2.2.2.7}$$

Using equations (2.2.2.4) – (2.2.2.7), the total cost $K(T_1,T)$ of an inventory system per time unit is given by

$$K(T_1,T) = CD + OC + IHC + SC$$

$$= \frac{CR}{T}\left[\sum_{n=0}^{\infty} \frac{\alpha^n T_1^{n\beta+1}}{n!(n\beta+1)} - T\right] + \frac{A}{T} + \frac{hR}{2T}\sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta+1)^2} T_1^{2(n\beta+1)} + \frac{\pi R}{2T}(T - T_1)^2$$

(2.2.2.8)

Here, T and T_1 are decision variables. The optimum value of T_1 and T can be obtained by solving

$$\frac{\partial K(T_1,T)}{\partial T} = 0 \text{; i.e.}$$

$$-\frac{CR}{T^2}\left[\sum_{n=0}^{\infty}\frac{\alpha^n T_1^{n\beta+1}}{n!(n\beta+1)}-T_1\right]-\frac{A}{T^2}-\frac{hR}{2T^2}\left[\sum_{n=0}^{\infty}\frac{(-1)^n\alpha^{2n}}{(n!)^2(n\beta+1)^2}T_1^{2(n\beta+1)}\right]$$
$$-\frac{\pi R T_1^2}{2T^2}+\frac{\pi R}{2}=0 \qquad (2.2.2.9)$$

and

$$\frac{\partial K(T_1,T)}{\partial T_1} = 0 \text{; i.e.}$$

$$CR\left[\sum_{n=0}^{\infty}\frac{\alpha^n T_1^{n\beta}}{n!}-1\right]+hR\left[\sum_{n=0}^{\infty}\frac{(-1)^n\alpha^{2n}}{(n!)^2(n\beta+1)}T_1^{2n\beta+1}\right]-\pi R(T-T_1)=0 \qquad (2.2.2.10)$$

simultaneously using suitable iterative method. The obtained values of T_1 and T will minimize the total cost of an inventory system provided

$$\frac{\partial^2 K(T_1,T)}{\partial T_1^2}\cdot\frac{\partial^2 K(T_1,T)}{\partial T^2}-\left[\frac{\partial^2 K(T_1,T)}{\partial T\,\partial T_1}\right]^2>0$$

2.2.3 Special Cases :

1) Shah's (1977) model can be obtained by putting $\alpha=\theta$ and $\beta=1$ in the above model.

$$Q=R\sum_{n=0}^{\infty}\frac{\theta^n T_1^{n+1}}{n!(n+1)}$$

$$K(T)=\frac{CR}{T}\left[T_1+\frac{\theta T_1^2}{2}-2T\right]+\frac{A}{T}+\frac{hR}{2T}\left[T_1^2-\frac{\theta^2 T_1^4}{4}\right]+\frac{\pi R(T-T_1)^2}{2T}$$

2.2.4. Assertions :

1. Total cost of an inventory system per time unit will increase with respect to the scale parameter α.

Proof :

$$\frac{dK}{d\alpha}=\frac{CR}{T}\left[\sum_{n=0}^{\infty}\frac{n\alpha^{n-1}T_1^{n\beta+1}}{n!(n\beta+1)}\right]+\frac{CIR}{2T}\left[\sum_{n=0}^{\infty}\frac{(-1)^n 2n\alpha^{2n-1}}{(n!)^2(n\beta+1)^2}T_1^{2(n\beta+1)}\right]>0, \quad \forall\ T,\ T_1$$

2. Total cost of an inventory system per time unit increases with respect to the ordering cost per order.

Proof : $$\frac{dK}{dA}=\frac{1}{T}>0 \quad \forall\ T$$

3. Increase in demand increases total cost of an inventory system per time unit.

Proof :

$$\frac{dK}{dR}=\frac{C}{T}\left[\sum_{n=0}^{\infty}\frac{\alpha^n T_1^{n\beta+1}}{n!(n\beta+1)}-T_1\right]+\frac{CI}{2T}\left[\sum_{n=0}^{\infty}\frac{(-1)^n\alpha^{2n}}{(n!)^2(n\beta+1)^2}T_1^{2(n\beta+1)}\right]+\frac{\pi}{2T}(T-T_1)^2>0, \quad \forall\ T,\ T_1$$

2.2.5 Numerical Example and observations :

Consider an inventory system with following parameters in appropriate units.

$$[\,C, I, R, A, \pi, \alpha, \beta\,]=[20, 12\%, 1000, 150, 30, 0.02, 1.5]$$

In the following table, the effect of various parameters on optimum time for having positive inventory, optimum cycle time, optimal procurement quantity, optimum positive stock and minimum cost of an inventory system is studied.

Variable		T_1	T	Q_1	K
C	20	0.92	1.174	933.33	1899.88
	25	0.91	1.206	918.70	2366.75
	30	0.89	1.236	905.07	2826.46
	35	0.88	1.263	892.27	3275.79
	40	0.87	1.288	880.19	3712.86
A	150	0.9267	1.174	933.33	1899.88
	200	0.9268	1.175	933.42	1949.30
	250	0.9268	1.177	933.51	1998.64
	300	0.9269	1.178	933.60	2047.89
	350	0.9270	1.180	933.69	2097.05
π	30	0.9267	1.174	933.33	1899.88
	35	0.9350	1.153	942.19	1879.02
	40	0.9422	1.137	949.14	1858.66
	45	0.9477	1.124	954.74	1839.52
	50	0.9522	1.113	959.35	1821.84
R	1000	0.926723	1.174	933.33	1899.88
	2000	0.926590	1.1717	1866.40	3651.19
	3000	0.926545	1.1709	2799.46	5402.36
	4000	0.926523	1.1706	3732.53	7153.50
	5000	0.926509	1.7103	4665.59	8904.50
α	0.02	0.9267	1.174	933.33	1899.88
	0.03	0.9216	1.167	931.45	1944.33
	0.04	0.9166	1.160	929.50	1987.99
	0.05	0.9116	1.153	927.49	2030.88
	0.06	0.9066	1.146	925.42	2073.01
β	1.5	0.9267	1.174	933.33	1899.88
	2.0	0.9246	1.133	923.95	1707.59
	2.5	0.9232	1.103	927.52	1584.17
	3.0	0.9221	1.081	925.72	1500.89
	3.5	0.9213	1.063	924.37	1442.49

Observations :

- Increase in purchase cost reduces cycle time of positive stock, increases optimum cycle time and total cost of an inventory system.
- Increase in ordering cost increases optimum cycle time, positive stock and total cost of an inventory system.
- Increase in shortage cost increases time of positive stock, and reduces total cycle time and total cost of an inventory system.
- Total cost increases significantly with increase in demand rate because procurement quantity increases drastically.
- Increase in deterioration rate (α) reduces time of positive stock and cycle time while increases total cost of an inventory system.
- Increase in shape parameter (β) reduces all decision variables and total cost of an inventory system.

2.3 Conclusion :

In this chapter two models have been proposed and formulated. An EOQ model has been derived in section 2.1 and an order level lot-size inventory model has been derived in section 2.2 when units in the inventory system are subject to time dependent deterioration. The analytic proofs are given to support each assertion of parameter dependence in both the models. The model in section 2.2 will reduce to the model in section 2.1 when there are no shortages i.e when $T_1 = 0$.

CHAPTER 3

INVENTORY MODELS WITH TIME DEPENDENT DETERIORATION OF UNITS UNDER CONDITIONS OF PERMISSIBLE DELAY IN PAYMENTS

3.0 Introduction.

In this chapter, three inventory models have been proposed. The mathematical models are developed to determine the optimal ordering policy for variable deterioration of units in an inventory system under permissible delay in payments.

In the models in **Section 3.1** and **3.2** the following two scenarios are discussed:

Scenario-1: When permissible delay period in payments is less than the cycle time

Scenario-2: When permissible delay period in payments is greater than the cycle time

Section 3.3 deals with an inventory model in which the supplier provides a cash discount as well as a credit period to the customer. It deals with the following four cases.

Case 1. The payment is paid at M_1 to get a cash discount and cycle time $T \geq M_1$

Case 2. The customer pays in full at M_1 to get a cash discount but cycle time $T<M_1$

Case 3. The payment is paid at time M to the permissible credit and cycle time $T \geq M$

Case 4. The customer pays in full at M and cycle time $T<M$

where M_1 : the period of cash discount

and M : the period of permissible delay

3.1 A LOT-SIZE MODEL WITH VARIABLE DETERIORATION RATE UNDER SUPPLIER CREDITS

This section deals with a lot-size model with variable deterioration rate under supplier credits. The model is developed under assumption of instantaneous and infinite replenishments and no shortages. The deterioration of items in the inventory follows the Weibull density function. The Newton-Raphson method has been used to find the optimum solutions. An easy-to-use algorithm to find the solution is given. Sensitivity analysis of the optimal solution with respect to the parameters of the system is carried out.

3.1.1 Assumptions and Notations

The following additional notations and assumptions other than those given in A.1 and N.1 are used to derive the proposed model

Assumptions :

- Shortages are not allowed.
- The distribution of the time for deterioration of units is as given in 1.3.1 earlier.

Q is a decision variable.

M is the permissible delay payment time.

3.1.2 Mathematical Formulation :

The model has two scenarios :

Scenario I : when permissible delay period 'M' in payments is less than the cycle time T;

Scenario II : when permissible delay period 'M' in payments is greater than the cycle time T.

In the first scenario, if the customer does not pay the supplier by time M, then he can incur an interest for the outstanding balance. In the second case, the customer would not only be able to use all the product he bought and get the revenue for that but also he would be able to earn the interest until the time he has to settle the account.

Let $Q(t)$, $0 \le t \le T$ be on-hand inventory of units at time t. The instantaneous state of $Q(t)$ for any instant of time follows the differential equation.

$$\frac{dQ(t)}{dt} + \theta(t)Q(t) = -R \qquad 0 \le t \le T \tag{3.1.2.1}$$

With initial condition $Q(0) = Q$ and $Q(T) = 0$

Equation (3.1.2.1) with given condition has solution.

$$Q(t) = \mathrm{Re}^{-\alpha t^{\beta}} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!(n\beta+1)}(T^{n\beta+1} - t^{n\beta+1}) \qquad 0 \le t \le T$$

$$Q(0) = Q = R \sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)} \tag{3.1.2.2}$$

The total demand during cycle time T is RT. Hence, the number of units deteriorated during $[0, T]$ is given by

$$D(T) = Q - RT$$

$$= R[\sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)} - T]$$

The cost due to deterioration / time unit is

$$CD = \frac{CD(T)}{T} = \frac{CR}{T}[\sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)} - T] \tag{3.1.2.3}$$

The ordering cost per time unit

$$OC = \frac{A}{T} \tag{3.1.2.4}$$

The inventory holding cost per time unit is

$$IHC = \frac{CI}{T}\int_0^T Q(t)dt$$

$$= \frac{CIR}{2T} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta+1)^2} T^{2(n\beta+1)} \tag{3.1.2.5}$$

Scenario I : $M \leq T$ (permissible delay period is less than the cycle time T)

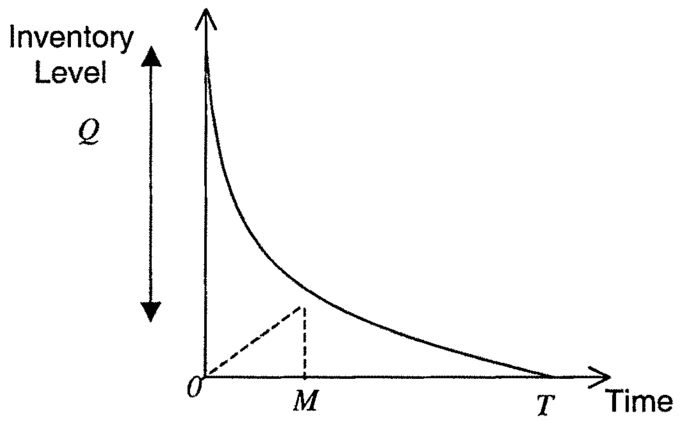


Fig. 3.1.1

Here the permissible payment period ends on or before the inventory depletes to zero. As a result, the variable cost consists of the sum of the ordering cost, inventory holding cost, cost due to deterioration of units and the interest charged minus the interest earned.

The interest payable per time unit is :

$$IC = \frac{CI_c R}{T} \int_M^T Q(t)dt$$

$$= \frac{CI_c R}{T} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta+1)^2} [\frac{T^{2(n\beta+1)}}{2} - M^{n\beta+1}(T^{n\beta+1} - \frac{M^{n\beta+1}}{2})] \qquad (3.1.2.6)$$

The interest earned per time unit is

$$IE = \frac{CI_e}{T} \int_0^M Rtdt = \frac{CI_e RM^2}{2T} \qquad (3.1.2.7)$$

Using equations (3.1.2.3) - (3.1.2.7), the total variable cost per time unit $K_1(T)$ is

$K_1(T) = OC + CD + IHC + IC - IE$

$$= \frac{A}{T} + \frac{CR}{T}\Big[\sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)} - T\Big] + \frac{CIR}{2T} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta+1)^2} T^{2(n\beta+1)}$$

$$+ \frac{CI_c R}{T} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta+1)^2}\Big[\frac{T^{2(n\beta+1)}}{2} - M^{n\beta+1}(T^{n\beta+1} - \frac{M^{n\beta+1}}{2})\Big]$$

$$- \frac{CI_e R}{T}(M - \frac{T}{2}) \qquad (3.1.2.8)$$

The optimal value of $T = T_1^{\circ}$ can be obtained by solving $\frac{dK_1(T)}{dT} = 0$

$$\frac{dK_1(T)}{dT} = -\frac{A}{T^2} + \frac{CR}{T}\Big[\sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta}}{n!} - 1\Big] - \frac{CR}{T^2}\Big[\sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)} - T\Big]$$

$$+ \frac{CIR}{T}\Big[\sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta+1)} T^{2n\beta+1}\Big] - \frac{CIR}{T^2}\Big[\sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta+1)^2} T^{2(n\beta+1)}\Big]$$

$$+ \frac{CI_e RM^2}{2T^2} + \frac{CI_c R}{T}\Big[\sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta+1)^2}\Big\{\frac{(n\beta+1)T^{2n\beta+1}}{2} - M^{n\beta+1}(n\beta+1)T^{n\beta}\Big\}\Big]$$

$$- \frac{CI_c R}{T^2}\Big[\sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta+1)^2}\Big\{\frac{T^{2(n\beta+1)}}{2} - M^{n\beta+1}(T^{n\beta+1} - \frac{M^{n\beta+1}}{2}\Big\}\Big] = 0$$

$$(3.1.2.9)$$

by suitable numerical method . The cycle time $T = T_1^{\circ}$ obtained by solving the equation (3.1.2.9), minimizes the total cost because $\frac{d^2K_1(T)}{dT^2} > 0$

$$\frac{\partial^2 K_1(T)}{\partial T^2} = \frac{2A}{T^3} + \frac{CR}{T}[\sum_{n=0}^{\infty} \frac{\alpha^n n\beta T^{n\beta-1}}{n!}] - \frac{2CR}{T^2}[\sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta}}{n!} - 1]$$

$$+\frac{2CR}{T^3}[\sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)} - T] + \frac{CIR}{2T}[\sum_{n=0}^{\infty} \frac{(-1)^n 2\alpha^{2n}}{(n!)^2(n\beta+1)}(2n\beta+1)T^{2n\beta}]$$

$$-\frac{3CIR}{2T^2}[\sum_{n=0}^{\infty} \frac{(-1)^n 2\alpha^{2n}}{(n!)^2(n\beta+1)} T^{2n\beta+1}] + \frac{2CIR}{T^3}[\sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)(n\beta+1)^2} T^{2(n\beta+1)}]$$

$$-\frac{CI_e RM^2}{T^3} + \frac{CI_C R}{T}[\sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2(n\beta+1)}\{\frac{(2n\beta+1)T^{2n\beta}}{2} - M^{n\beta+1}(n\beta)T^{n\beta-1}\}]$$

$$-\frac{2CI_C R}{T^2}[\sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2(n\beta+1)}\{\frac{T^{2(n\beta+1)}}{2} - M^{n\beta+1}T^{n\beta}\}]$$

$$+\frac{2CI_C R}{T^3}[\sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2(n\beta+1)^2}\{\frac{T^{2(n\beta+1)}}{2} - M^{n\beta+1}(T^{n\beta+1} - \frac{M^{n\beta+1}}{2})\}]$$

Scenario II : $T < M$ (permissible delay period is greater than the cycle time T)

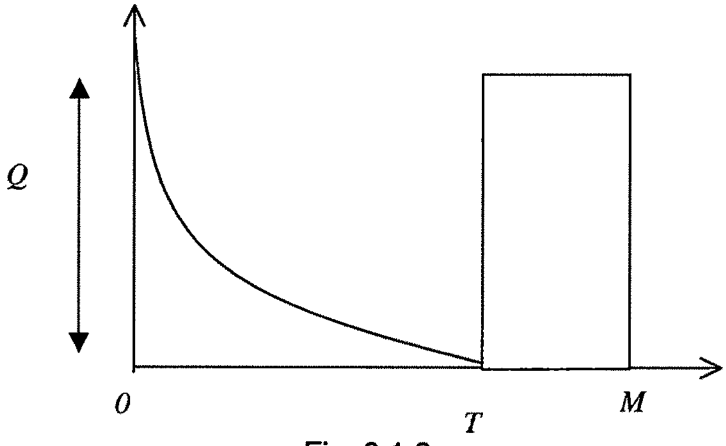


Fig. 3.1.2

Here the payment is made after the permissible delay period. So there is no interest charged. The interest earned per time unit is

$$IE = \frac{CI_e}{T}[\int_0^T Rtdt + RT(M-T)]$$

$$= \frac{CI_e R}{T}(M - \frac{T}{2}) \qquad (3.1.2.10)$$

Hence, the total variable cost, $K_2(T)$ per time unit is

$$K_2(T) = OC + CD + IHC - IE$$

$$= \frac{A}{T} + \frac{CR}{T}[\sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)} - T] + \frac{CIR}{2T} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta+1)^2} T^{2(n\beta+1)}$$

$$- \frac{CI_e R}{T}(M - \frac{T}{2}) \qquad (3.1.2.11)$$

The optimal value of $T = T_2^{\circ}$ can be obtained by solving $\frac{dK_2(T)}{dT} = 0$

i.e.

$$-\frac{A}{T^2} + \frac{CR}{T}[\sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta}}{n!} - 1] - \frac{CR}{T^2}[\sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)} - T]$$

$$+ \frac{CIR}{2T}[\sum_{n=0}^{\infty} \frac{(-1)^n 2\alpha^{2n}}{(n!)^2 (n\beta+1)} T^{2n\beta+1}] - \frac{CIR}{2T^2}[\sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta+1)^2} T^{2(n\beta+1)}]$$

$$+ \frac{CI_e R}{2T} + \frac{CI_e R}{T^2}(M - \frac{T}{2}) = 0 \qquad (3.1.2.12)$$

by suitable numerical method. The cycle time $T = T_2^{\circ}$ obtained by solving the equation (3.1.12), minimizes the total cost because $\frac{d^2K_2(T)}{dT^2} > 0$

$$\frac{\partial^2 K_2(T)}{\partial T^2}=\frac{2A}{T^3}+\frac{CR}{T}[\sum_{n=0}^{\infty}\frac{\alpha^n n\beta T^{n\beta-1}}{n!}]-\frac{2CR}{T^2}[\sum_{n=0}^{\infty}\frac{\alpha^n T^{n\beta}}{n!}-1]$$

$$+\frac{2CR}{T^3}[\sum_{n=0}^{\infty}\frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)}-T]+\frac{CIR}{2T}[\sum_{n=0}^{\infty}\frac{(-1)^n 2\alpha^{2n}}{(n!)^2(n\beta+1)^2}(2n\beta+1)T^{2n\beta}]$$

$$-\frac{3CIR}{2T^2}[\sum_{n=0}^{\infty}\frac{(-1)^n 2\alpha^{2n}}{(n!)^2(n\beta+1)^2}T^{2n\beta+1}]+\frac{CIR}{T^3}[\sum_{n=0}^{\infty}\frac{(-1)^n\alpha^{2n}}{(n!)(n\beta+1)^2}T^{2(n\beta+1)}]$$

$$-\frac{CI_eR}{2T^2}-\frac{CI_eR}{T^3}(M-\frac{T}{2})-\frac{CI_eR}{2T^2}$$

Algorithm : The optimum cycle time

$$T_0=\begin{cases}T_1^{\circ} & M\le T\\ T_2^{\circ} & M>T\end{cases}$$

Hence,

$$Q_0=\begin{cases}Q_1^{\circ} & M\le T\\ Q_2^{\circ} & M>T\end{cases}$$

and total cost of an inventory system

$$K(T_0)=\begin{cases}K_1(T_1^{\circ}) & M\le T\\ K_2(T_2^{\circ}) & M>T\end{cases}$$

3.1.3 Numerical example and observations :

Consider an inventory system with following parameters in appropriate units.

$[A, R, C, I, I_c, I_e, \alpha, \beta, M]$ = [250, 2000, 20, 0.10, 0.12, 0.15, 0.02,1.5,15/365]

Variable		T	Q	K
R	2000	0.2191	438.73	2027.37
	2500	0.1963	491.27	2233.42
	3000	0.1795	538.86	2414.08
	3500	0.1664	582.72	2575.52
	4000	0.1558	623.62	2721.78
α	0.02	0.2191	438.73	2027.37
	0.03	0.2169	434.33	2043.66
	0.04	0.2147	430.10	2059.70
	0.05	0.2126	426.04	2075.50
	0.06	0.2105	422.12	2091.07
β	1.5	0.2191	438.73	2027.37
	2	0.2214	443.06	2007.22
	2.5	0.2227	445.51	1999.38
	3	0.2233	446.80	1996.23
	3.5	0.2237	447.47	1994.94
I	0.12	0.2113	422.98	2113.44
	0.13	0.2076	415.71	2155.34
	0.14	0.2042	408.80	2196.54
	0.15	0.2009	402.22	2237.06
C	20	0.2191	438.73	2027.37
	25	0.1963	393.01	2233.42
	30	0.1795	359.24	2414.08
	35	0.1664	332.98	2575.52
	40	0.1558	311.81	2721.78
A	250	0.2191	438.73	2027.37
	300	0.2397	480.03	2245.24
	350	0.2587	517.97	2445.84
	400	0.2763	553.27	2632.74
	450	0.2928	586.39	2808.44
M	15/365	0.2191	438.73	2027.37
	30/365	0.2204	441.35	1795.16
	45/365	0.2226	445.68	1571.41
I_C	0.15	0.2191	438.73	2027.37
	0.20	0.2016	403.56	2163.25
	0.25	0.1878	375.97	2284.23
I_e	0.12	0.2191	438.73	2027.37
	0.15	0.2187	437.85	2022.74
	0.20	0.2180	436.38	2015.01

Observations:

- Increase in the deterioration rate (α) decreases the cycle time and the procurement quantity and increases the total cost of the inventory system.
- Increase in the demand rate (R) decreases the cycle time and increases the procurement quantity and the total cost of the inventory system significantly.
- Increase in the shape parameter (β) increases the cycle time and the procurement quantity and decreases the total cost of the inventory system.
- Increase in the purchase cost (C) decreases the cycle time and the procurement quantity while increases the total cost of the inventory system.
- Increase in the ordering cost (A) increases the cycle time, the procurement quantity and increases the total cost of the inventory system significantly.
- Increase in the carrying charge fraction (I) per unit per annum reduces the cycle time and the procurement quantity and increases the total cost of the inventory system.
- Increase in the delay period (M) increases the cycle time and the procurement quantity while reduces the total cost of the inventory system significantly.
- Increase in the interest charged (I_c) reduces the cycle time and the purchase quantity while increases the total cost of the inventory system.
- Increase in the interest earned (I_e) reduces the cycle time, procurement quantity and the total cost of an inventory system.

3.2 AN ORDER LEVEL LOT–SIZE MODEL WITH TIME DEPENDENT DETERIORATION AND PERMISSIBLE DELAY IN PAYMENTS

The model in this section is an extension of the previous model. Shortages are allowed here. The rest of the assumptions remain same as that of the model in section 3.1. The mathematical methodology to find the solution of the equations is explained in detail and a special case is also discussed. Sensitivity analysis is carried out at the end and a few directions for future research are discussed.

3.2.1 Assumptions and Notations:

The following additional assumptions and notations other than those given in A.1 and N.1 are used to formulate the proposed model.

Shortages are allowed.

The distribution of the time for deterioration of units is as given in 1.3.1

R : The known demand rate (units per time unit).

Q_1 : Quantity consumed during time T_1.

T_1 : length of the period with positive stock of the items in the inventory.

3.2.2 The Mathematical formulation :

The model has two scenarios

Scenario I : When permissible delay 'M' in payments is less than the period having inventory stock in hand T_1.

Scenario II : When permissible delay; M, in payments is greater than the period having inventory stock in hand T_1.

In the first case, if the customer does not pay the supplier by time M, then he can incur an interest for the outstanding balance. In the second case, the customer would not only be able to use all the product he bought and get the revenue for that but also he would be able to earn interest on that revenue until the time he has to settle the account. There could be some other situations involving payment of dues to the supplier either during the inventory in stock or shortages. All these scenarios are discussed in subsequent sections.

Let $Q(t)$ be the inventory level at time t. Depletion of inventory occurs due to the simultaneous demand and deterioration of units. The deterioration of units occurs during time period $(0, T_1)$ and shortages occur during time interval (T_1, T). (See fig. 3.2.1)

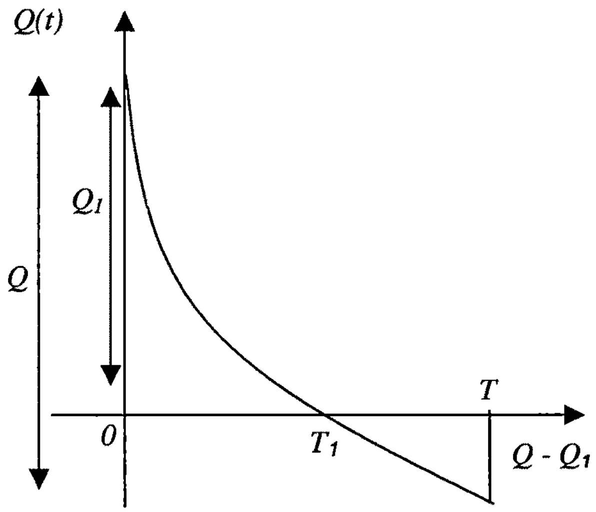


Fig. 3.2.1

The differential equation governing instantaneous state of units in inventory at time t $(0 \le t \le T)$ is given by

$$\frac{dQ(t)}{dt} + \theta(t)Q(t) = -R \qquad 0 \le t \le T_1 \qquad (3.2.2.1)$$

$$\frac{dQ(t)}{dt} = -R \qquad T_1 \le t \le T$$

where at time $t = 0$, $Q(0) = Q$.

The solution of eq. (3.2.2.1) is given by

$$Q(t) = \text{R}e^{-\alpha t^{\beta}} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!(n\beta+1)}(T_1^{n\beta+1} - t^{n\beta+1}) \qquad 0 \le t \le T_1 \qquad (3.2.2.2)$$

Then

$$Q(0) = Q = R \sum_{n=0}^{\infty} \frac{\alpha^n T_1^{n\beta+1}}{n!(n\beta+1)} \qquad (3.2.2.3)$$

The demand during T_1 is RT_1.

The number of units deteriorated during one cycle is given by

$$D(T) = Q - RT_1$$

$$= R \left[\sum_{n=0}^{\infty} \frac{\alpha^n T_1^{n\beta+1}}{n!(n\beta+1)} - T_1 \right] \qquad (3.2.2.4)$$

Since the shortages are allowed in the present mathematical model, there are two cases for payment to be made at time M. These cases are :

1. Payment at or before the total depletion of inventory; i.e. $(M \le T_1 < T)$.

2. Payment after depletion i.e. $T_1 < M$.

Case 1. $M \le T_1 < T$.

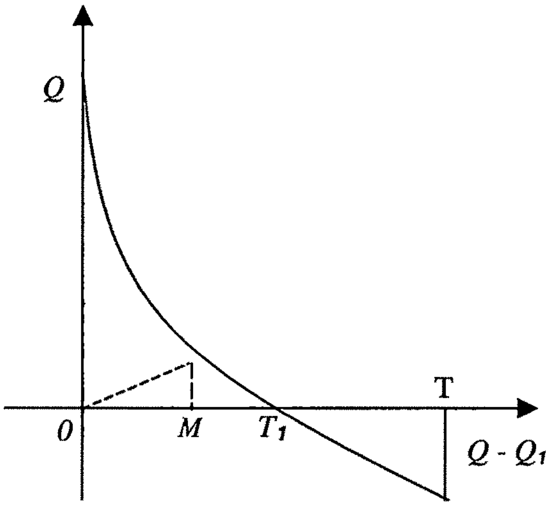


Fig. 3.2.2

Here the permissible payment period ends on or before the inventory depleted completely to zero. As a result, the variable cost consists of the sum of the ordering cost, inventory holding cost, shortage cost, cost due to deterioration of units and the interest charged minus the interest earned.

These costs are as under.

The ordering cost, $OC = A$ (3.2.2.5)

The cost of deterioration *(CD)* incurred to *D(T)* units of material per cycle time *T* is given by

$$CD = CD(T)$$

$$= CR\ [\sum_{n=0}^{\infty} \frac{\alpha^n T_1^{n\beta+1}}{n!(n\beta+1)} - T_1] \qquad (3.2.2.6)$$

The inventory holding cost; *IHC* is

$$IHC = CI \int_0^{T_1} Q(t)dt$$

$$= \frac{CIR}{2} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta+1)^2} T_1^{2(n\beta+1)} \qquad (3.2.2.7)$$

The interest charged per cycle for the inventory not being sold after the due date *M* is

$$IC = CI_c \int_M^{T_1} Q(t)dt$$

$$= CI_c R \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^n}{(n!)(n\beta+1)} [T^{n\beta+1}(T_1^{n\beta+1} - M^{n\beta+1}) - \frac{1}{2}(T_1^{2(n\beta+1)} - M^{2(n\beta+1)})]$$

(3.2.2.8)

Interest earned per cycle, *IE*, during the positive inventory is given by

$$IE = CI_e \int_0^M Rtdt = \frac{CI_e RM^2}{2} \qquad (3.2.2.9)$$

The backordered cost, *SC*, per cycle is given by

$$SC = \pi \int_0^{T-T_1} Rtdt = \frac{\pi R(T - T_1)^2}{2} \qquad (3.2.2.10)$$

Hence, the total variable cost, $K_1(T_1,T)$ per time unit is

$$K_1(T_1,T)=\frac{1}{T}(OC+CD+IHC+IC+SC-IE)$$

$$=\frac{1}{T}\{A+CR(\sum_{n=0}^{\infty}\frac{\alpha^n T_1^{n\beta+1}}{n!(n\beta+1)}-T_1)+\frac{CIR}{2}\sum_{n=0}^{\infty}\frac{(-1)^n\alpha^{2n}}{(n!)^2(n\beta+1)^2}T_1^{2(n\beta+1)}$$

$$+CI_cR\sum_{n=0}^{\infty}\frac{(-1)^n\alpha^n}{(n!)(n\beta+1)}[T^{n\beta+1}(T_1^{n\beta+1}-M^{n\beta+1})-\frac{1}{2}(T_1^{2(n\beta+1)}-M^{2(n\beta+1)})]$$

$$+\frac{\pi R(T-T_1)^2}{2}-\frac{CI_eRM^2}{2}\} \quad (3.2.2.11)$$

To evaluate the nature of the total cost function in (3.2.2.11), it is to establish whether the function is convex or not. Since $K_1\ (T_1,T)$ involves higher order and summation, it is not easy to evaluate the Hessians in closed – form to conclude about its positive definiteness directly. The total cost $K_1\ (T_1,T)$ is evaluated over certain range of T_1 and T with different sets of inventory parametric values. Therefore, the values of T and T_1 which minimize $K_1(T_1,T)$ can be obtained by simultaneously solving $\partial K_1(T_1,T)/\ \partial T_1 = 0$ and $\partial K_1(T_1,T)/\ \partial T = 0$ within the stated ranges. These two partial differential equations lead to the equations (3.2.2.12) and (3.2.2.13) as shown below.

$$\frac{\partial K_1(T_1,T)}{\partial T_1}=0$$

$$\therefore\ \frac{CR}{T}[\sum_{n=0}^{\infty}\frac{\alpha^n T_1^{n\beta}}{n!}-1]+\frac{CIR}{2T}[\sum_{n=0}^{\infty}\frac{(-1)^n 2\alpha^n}{(n!)^2(n\beta+1)}T_1^{n\beta+1}]-\frac{\pi R}{T}(T-T_1)$$

$$+\frac{CI_cR}{T}\sum_{n=0}^{\infty}\frac{(-1)^n\alpha^n}{n!}T_1^{n\beta}(T^{n\beta+1}-T_1^{n\beta+1})-\frac{CI_eRT_1}{T}=0 \quad (3.2.2.12)$$

and

$$\frac{\partial K_1(T_1,T)}{\partial T}=0$$

$$\therefore\ -CR[\sum_{n=0}^{\infty}\frac{\alpha^n T_1^{n\beta+1}}{n!(n\beta+1)}-T_1]-A-\frac{CIR}{2}[\sum_{n=0}^{\infty}\frac{(-1)^n\alpha^{2n}}{(n!)^2(n\beta+1)^2}T_1^{2(n\beta+1)}]$$

$$-\frac{\pi R(T-T_1)^2}{2}+\pi R(T-T_1)T$$

$$-CI_C R\sum_{n=0}^{\infty}\frac{(-1)^n\alpha^n}{(n!)}[\frac{T^{n\beta+1}}{(n\beta+1)}(T_1^{n\beta+1}-M^{n\beta+1})-\frac{1}{2(n\beta+1)}(T_1^{2(n\beta+1)}-M^{2(n\beta+1)})]$$

$$+CI_C RT\sum_{n=0}^{\infty}\frac{(-1)^n\alpha^n}{(n!)}T^{n\beta}(T_1^{n\beta+1}-M^{n\beta+1})+\frac{CI_e RM^2}{2T^2}=0 \qquad (3.2.2.13)$$

Since equations (3.2.2.12) and (3.2.2.13) are functions of T_1 and T, and the convexity of none of the functions is assured in general, the iterative search approach must be used simultaneously to obtain pragmatic solutions for T_1 and T. When the initial value of T_1 within certain feasible range is assumed in (3.2.2.12), the solution of T is immediately known. If the value of T obtained from (3.2.2.12) is then used as an initial value in (3.2.2.13), the value of T_1 is also known by a one dimensional iterative search procedure. This value of T_1 may not be equal to the value of T_1 obtained earlier from (3.2.2.12). The process of switching between the equations is repeated until two consecutive iterations give same values of T_1 and T. Once T_1 and T are obtained, the optimal ordering quantity and total cost of an inventory system is calculated easily.

A Special case : $T_1 = M$.

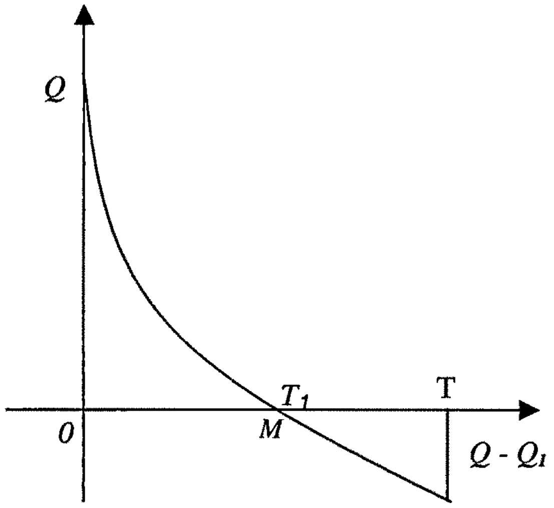


Fig. 3.2.3

If payment is made at the time $T_1 = M$, the ordering cost remains the same as before, and also the deterioration cost, inventory holding cost, interest earned and the shortage cost remain the same as in the earlier case. Since the payment is made when it is due at time T_1, the interest charged, IC, is zero Therefore, $\partial K_2(M,T)/\partial M = 0$ after replacing T_1 with M in (3.2.2.12) and $\partial K_2(M,T)/\partial T = 0$ in (3.2.2.13) as given (3.2.2.14) & (3.2.2.15) shown below.

$$K_2(M,T) = \frac{CR}{T}[\sum_{n=0}^{\infty} \frac{\alpha^n M^{n\beta+1}}{n!(n\beta+1)} - M] + \frac{A}{T} + \frac{CIR}{2T}[\sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta+1)^2} M^{2(n\beta+1)}] + \frac{\pi R(T-M)^2}{2T} - \frac{CI_e RM^2}{2T}$$

$$\frac{\partial K_2(M,T)}{\partial M} = 0$$

$$\therefore \frac{CR}{T}[\sum_{n=0}^{\infty} \frac{\alpha^n M^{n\beta}}{n!} - 1] + \frac{CIR}{2T}[\sum_{n=0}^{\infty} \frac{2(-1)^n \alpha^n}{(n!)^2 (n\beta+1)^2} M^{2(n\beta+1)}] - \frac{\pi R(T-M)}{T} + \frac{CI_e RM}{T} = 0 \quad (3.2.2.14)$$

$$\frac{\partial K_2(M,T)}{\partial T}=0$$

$$\therefore\ -CR[\sum_{n=0}^{\infty}\frac{\alpha^n M^{n\beta+1}}{n!(n\beta+1)}-M]-A-\frac{CIR}{2}[\sum_{n=0}^{\infty}\frac{(-1)^n\alpha^{2n}}{(n!)^2(n\beta+1)^2}M^{2(n\beta+1)}]$$

$$-\frac{\pi R(T-M)^2}{2}+\pi R(T-M)T+\frac{CI_eRM^2}{2}=0 \qquad (3.2.2.15)$$

Case 2: $T_1 < M$ (After – depletion payment)

The deterioration cost CD; the inventory holding cost, IHC, and the shortage cost, SC, per cycle are the same as in the earlier case. The interest charged per cycle $IC = 0$ when $T_1 < M \leq T$ because the supplier can be paid in full at time M, the permissible delay period.

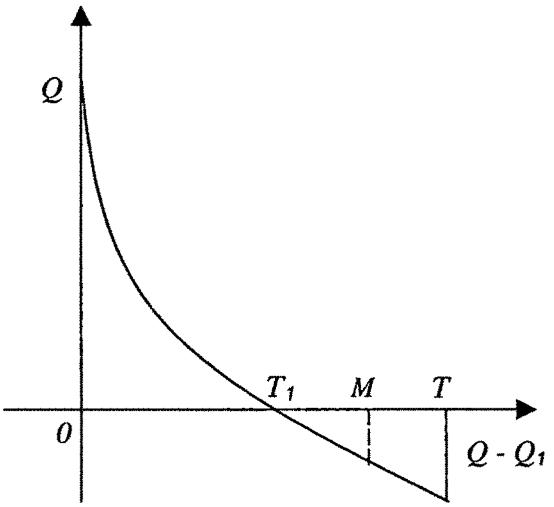


Fig. 3.2.4

The interest earned per cycle is the interest earned during the positive inventory period plus the interest earned from the cash invested during time period (T_1,M) after the inventory is exhausted at time T_1, and it is given by

$$IE=CI_e\int_0^{T_1}Rtdt+CRI_eT_1(M-T_1)$$

$$=CRI_eT_1(M-\frac{T_1}{2}) \qquad (3.2.2.16)$$

Incorporating these modifications in (3.2.2.11), the total variable cost per unit time, $K_3(T_1,T)$ is given by

$$K_3(T_1,T)=\frac{1}{T}(OC+CD+IHC+SC-IE)$$

$$=\frac{1}{T}\{A+CR(\sum_{n=0}^{\infty}\frac{\alpha^n T_1^{n\beta+1}}{n!(n\beta+1)}-T_1)+\frac{CIR}{2}\sum_{n=0}^{\infty}\frac{(-1)^n\alpha^{2n}}{(n!)^2(n\beta+1)^2}T_1^{2(n\beta+1)}$$

$$+\frac{\pi R(T-T_1)^2}{2}-CI_eRT_1(M-\frac{T_1}{2})\} \qquad (3.2.2.17)$$

As in case 1, the total cost is minimized when $\partial K_3(T_1,T)/\partial T_1 = 0$ (eq. 3.2.2.18) and $K_3(T_1,T)/\partial T = 0$ (eq. 3.2.2.19) as shown below.

$$\frac{\partial K_3(T_1,T)}{\partial T_1}=0$$

$$\therefore\ \frac{CR}{T}[\sum_{n=0}^{\infty}\frac{\alpha^n T_1^{n\beta}}{n!}-1]+\frac{CIR}{2T}[\sum_{n=0}^{\infty}\frac{(-1)^n 2\alpha^{2n}}{(n!)^2(n\beta+1)}T^{n\beta+1}]-\frac{\pi R(T-T_1)}{T}-CI_eR(M-T_1)=0 \qquad (3.2.2.18)$$

$$\frac{\partial K_3(T_1,T)}{\partial T}=0$$

$$\therefore\ -CR[\sum_{n=0}^{\infty}\frac{\alpha^n T_1^{n\beta+1}}{n!(n\beta+1)}-T_1]-A-\frac{CIR}{2}[\sum_{n=0}^{\infty}\frac{(-1)^n\alpha^{2n}}{(n!)^2(n\beta+1)^2}T_1^{2(n\beta+1)}]$$

$$-\frac{\pi R(T-T_1)^2}{2}+\pi R(T-T_1)T=0 \qquad (3.2.2.19)$$

Equations (3.2.2.18) and (3.2.2.19) need to be solved simultaneously for optimal values of T_1 and T as it is done in case 1.

3.2.3 Numerical Example and observations :

Consider, the inventory parametric values R = 2000 units / year, A = \$ 250 / order, I = 10% / unit / year, I_e = 12% /unit / year, I_c = 15% / unit / year, β = 1.5. Following the procedure given in above section, the economic ordering policies computed for different values of α, C and π are given in Tables 1 – 3.

Table 3.2.1 (Variation in α)

α	T	T_1	Q	Q_1	Q_2	K
0.02	0.254	0.187	508	374	134	3593.72
0.03	0.232	0.146	464	292	172	3495.57
0.04	0.217	0.123	434	246	188	3420.88

Table 3.2.2 (Variation in C)

C	T	T_1	Q	Q_1	Q_2	K
20	0.254	0.187	508	374	134	3593.72
25	0.231	0.161	462	322	140	4192.48
30	0.220	0.148	440	296	144	4865.34
35	0.218	0.132	436	264	172	5692.00
40	0.204	0.121	408	242	166	6339.54

Table 3.2.3 (Variation in π)

π	T	T_1	Q	Q_1	Q_2	K
30	0254	0.187	508	374	134	3593.72
35	0.242	0.193	484	386	98	3421.70
40	0.234	0.211	468	422	46	3254.37
45	0.223	0.219	446	438	8	3092.23

Observations :

- Increase in deterioration rate reduces optimal cycle time, as a result optimal procurement quantity decreases. With increase in deterioration there is a decrease in T_1 and Q_1. As a result shortages increase. Because we are procuring smaller quantities there is a decrease in total inventory cost of the system.

- Increase in the purchase cost, decreases the optimal cycle time, optimal procurement quantity Q, T_1, Q_1 and increases shortages and also increases the total cost of the inventory system.
- Increase in the unit backorder cost decreases optimal cycle time, optimal procurement quantity and total cost of an inventory system whereas there is increase in T_1 and Q_1.

3.3 AN EOQ MODEL FOR DETERIORATING ITEMS WITH TWO PARAMETER WEIBULL DISTRIBUTION DETERIORATION UNDER SUPPLIER CREDITS

In this section, an inventory model is developed for deteriorating items with two parameter Weibull distribution, in which the supplier provides both cash discount and credit period to the customer. Also, in the model, lead time is zero, replenishment is infinite and shortages are not allowed.

In the model the following four cases are discussed based on the provisions of discount and permissible delay periods.

Case 1. The payment is paid at M_1 to get a cash discount and cycle time $T \geq M_1$

Case 2. The customer pays in full at M_1 to get a cash discount but cycle time $T<M_1$

Case 3. The payment is paid at time M to the permissible credit and cycle time $T \geq M$

Case 4. The customer pays in full at M and cycle time $T<M$

The Taylor's series approximation is used to determine the mathematical results. A numerical example is provided to verify the results obtained in the market. A few directions for future research are also given.

3.3.1 Assumptions and Notations:

The mathematical model is derived with the following additional assumptions and notations other than those given in A.1 and N.1

- The demand for the item is constant during the cycle time.
- Shortages are not allowed.
- The distribution of the time for deterioration of units is as given in 1.3.1

Notations :

- = the cash discount rate, $0 < r < 1$.
- = The period of cash discount.

M = The period of permissible delay in settling account, with $M > M_1$.

$K(T)$ = The total relevant cost per year which consists of (a) ordering cost, (b) cost of deteriorating units, (c) inventory carrying cost (excluding interest charges), (d) cash discount earned if the payment is made at M_1, (e) cost of interest charges for unsold items after the permissible credit period M, minus (f) interest earned from sales revenue during the permissible delay period.

3.3.2 Mathematical Formulation:

The inventory level $Q(t)$ gradually decreases to meet demands and partly due to deterioration. Hence the instantaneous rate of inventory level at any instant of time t can be represented by the following differential equation

$$\frac{dQ(t)}{dt} + \theta(t)Q(t) = -R \qquad 0 \le t \le T \qquad (3.3.2.1)$$

with the boundary conditions $Q(0) = Q$ and $Q(T) = 0$. Then the solution of differential equation (3.3.2.1) is given by

$$Q(t) = Re^{-\alpha t^{\beta}} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!(n\beta+1)} (T^{n\beta+1} - t^{n\beta+1}) \qquad (3.3.2.2)$$

and the order quantity is

$$Q = R \sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)} \tag{3.3.2.3}$$

Total demand during one cycle is RT.

Hence, the number of units which deteriorate during a replenishment cycle is

$$D(T) = Q - RT$$

$$= R\left[\sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)} - T \right] \tag{3.3.2.4}$$

The total relevant cost per time unit consists of the following components.

(a) OC = Cost of placing an order = A/T (3.3.2.5)

(b) CD = Cost of deteriorated unit $= \frac{CR}{T}\left[\sum_{n=0}^{\infty} \frac{a^n T^{nb+1}}{n!(nb+1)} - T \right]$ (3.3.2.6)

(c) IHC = Cost of carrying inventory $= \frac{CIR}{2T} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta+1)^2} T^{2(n\beta+1)}$ (3.3.2.7)

Regarding cash discount, interests charged and earned, there are following four cases based on the customer's two choices (i.e. : pays at M_1 or M) and the length of cycle time T.

Case 1. The payment is paid at M_1 to get a cash discount and $T \geq M_1$ (fig. 3.3.1)

Case 2. The customer pays in full at M_1 to get a cash discount but $T<M_1$ (fig. 3.3.2)

Case 3. The payment is paid at time M to the permissible credit and $T \geq M$ (fig. 3.3.3)

Case 4. The customer pays in full at M and $T<M$ (fig. 3.3.4)

So next we derive interest earned and interest paid in each case.

Case 1 : $T \geq M_1$

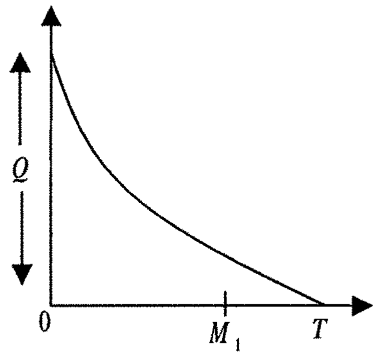


Fig. 3.3.1

Hence the payment is paid to be made at time M_1. Using (4), the cash discount per year is given by

$$CDC = \frac{rcQ}{T} = \frac{rcR}{T} \sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)} \tag{3.3.2.8}$$

The interest charged per year is

$$IC_1 = \frac{CI_c R}{T} \int_{M_1}^{T} Q(t)dt$$

$$= \frac{CI_c R}{T} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{n(\beta+1)}}{(n!)^2 (n\beta+1)^2} \left[\frac{T^{2(n\beta+1)}}{2} - M_1^{n\beta+1}\left(T^{n\beta+1} - \frac{M_1^{n\beta+1}}{2}\right) \right] \tag{3.3.2.9}$$

The interest charged per year is

$$IE_1 = \frac{PI_e}{T} \int_{0}^{M_1} Rtdt = \frac{PI_e RM_1^2}{2T} \tag{3.3.2.10}$$

From (3.3.2.5) – (3.3.2.10), the total relevant cost per year $K_1(T)$ is given by

$$K_1(T) = OC + CD + IHC + CDC + IC_1 - IE_1$$

$$=\frac{A}{T}+\frac{CR}{T}\left[\sum_{n=0}^{\infty}\frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)}-T\right]+\frac{CIR}{2T}\sum_{n=0}^{\infty}\frac{(-1)^n\alpha^{2n}}{(n!)^2(n\beta+1)^2}T^{2(n\beta+1)}+\frac{rcR}{T}\sum_{n=0}^{\infty}\frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)}$$

$$+\frac{CI_cR}{T}\sum_{n=0}^{\infty}\frac{(-1)^n\alpha^{n(\beta+1)}}{(n!)^2(n\beta+1)^2}\left[\frac{T^{2(n\beta+1)}}{2}-M_1^{n\beta+1}\left(T^{n\beta+1}-\frac{M_1^{n\beta+1}}{2}\right)\right]-\frac{PI_eRM_1^2}{2T}$$

(3.3.2.11)

Case 2 : $T < M_1$

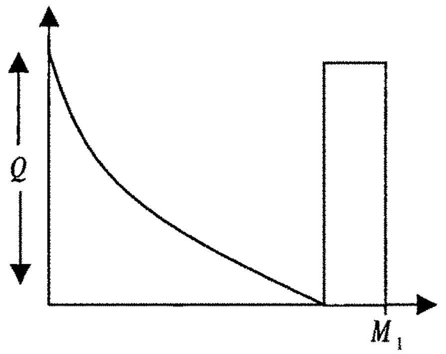


Fig. 3.3.2

In this case, the interest charges are zero, but the cash discount is same as that in case 1. The interest earned per year is

$$IE_2=\frac{PI_e}{T}\left[\int_0^T Rtdt-RT(M_1-T)\right]$$

$$=PI_eR(M_1-\frac{T}{2}) \qquad (3.3.2.12)$$

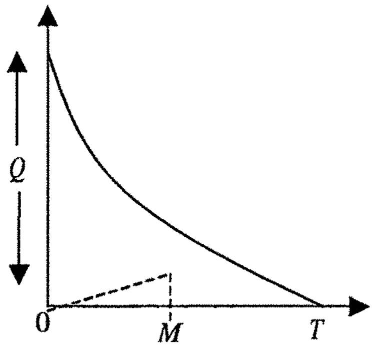
As a result, the total relevant cost per year $K_2(T)$ is

$$K_2(T)=OC+CD+IHC+CDC-IE_2$$

$$=\frac{A}{T}+\frac{CR}{T}\left[\sum_{n=0}^{\infty}\frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)}-T\right]+\frac{CIR}{2T}\sum_{n=0}^{\infty}\frac{(-1)^n\alpha^{2n}}{(n!)^2(n\beta+1)^2}T^{2(n\beta+1)}$$

$$+\frac{rcR}{T}\sum_{n=0}^{\infty}\frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)}-PI_eR(M_1-\frac{T}{2}) \qquad (3.3.2.13)$$

Case 3. $T \geq M$



(Fig. 3.3.3)

Here the payment is made at time M, there is no cash discount.

The interest payable per year is

$$IC_3 = \frac{CI_c}{T}\int_M^T Q(t)dt$$

$$= \frac{CI_cR}{T}\sum_{n=0}^{\infty}\frac{(-1)^n\alpha^{n(\beta+1)}}{(n!)^2(n\beta+1)^2}\left[\frac{T^{2(n\beta+1)}}{2} - M^{n\beta+1}\left(T^{n\beta+1} - \frac{M^{n\beta+1}}{2}\right)\right]$$

(3.3.2.14)

The interest earned per year is

$$IE_3 = \frac{PI_e}{T}\int_0^M Rtdt = \frac{PI_eRM^2}{2T} \tag{3.3.2.15}$$

Therefore, the total relevant cost per year $K_3(T)$ is

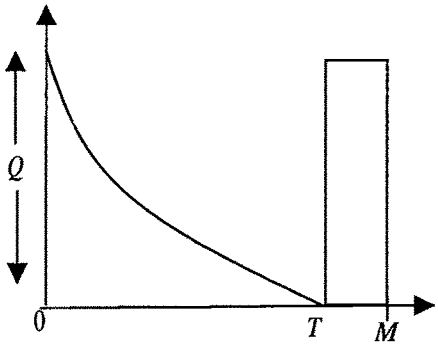
$$K_3(T) = OC + CD + IHC + IC_3 - IE_3$$

$$= \frac{A}{T} + \frac{CR}{T}\left[\sum_{n=0}^{\infty}\frac{\alpha^nT^{n\beta+1}}{n!(n\beta+1)} - T\right] + \frac{CIR}{2T}\sum_{n=0}^{\infty}\frac{(-1)^n\alpha^{2n}}{(n!)^2(n\beta+1)^2}T^{2(n\beta+1)}$$

$$+\frac{CI_cR}{T}\sum_{n=0}^{\infty}\frac{(-1)^n\alpha^{n(\beta+1)}}{(n!)^2(n\beta+1)^2}\left[\frac{T^{2(n\beta+1)}}{2} - M^{n\beta+1}\left(T^{n\beta+1} - \frac{M^{n\beta+1}}{2}\right)\right]$$

$$-\frac{PI_eRM^2}{2T} \tag{3.3.2.16}$$

Case 4. $T < M$



(Fig. 3.3.4)

In this case, the interest charged is zero. The interest earned per year is

$$IE_4 = \frac{CI_e}{T}[\int_0^T Rtdt + RT(M-T)]$$

$$= \frac{CI_eR}{T}[M - \frac{T}{2}] \qquad (3.3.2.17)$$

Hence the total relevant cost per year $K_4(T)$ is

$$K_4(T) = OC + CD + IHC - IE_4$$

$$= \frac{A}{T} + \frac{CR}{T}[\sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)} - T] + \frac{CIR}{2T}\sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2(n\beta+1)^2} T^{2(n\beta+1)}$$

$$- \frac{CI_eR}{T}[M - \frac{T}{2}] \qquad (3.3.2.18)$$

Theoretical Results :

Assuming, α - the rate deterioration to be very small, we get

$$K_1(T) = \frac{A}{T} + \frac{CR\alpha T^\beta}{\beta+1}(1+r) + CR(\frac{IT}{2}+r) + \frac{CI_cR}{T}[\frac{T^2}{2} - M_1(T - \frac{M_1}{2})]$$

$$- \frac{CI_cR\alpha^{\beta+1}}{T(\beta+1)^2}[\frac{T^{2(\beta+1)}}{2} - M_1^{\beta+1}(T^{\beta+1} - \frac{M_1^{\beta+1}}{2}] - \frac{PI_eRM_1^2}{2T} \qquad T \geq M_1$$

(3.3.2.19)

$$K_2(T)=\frac{A}{T}+\frac{CR\alpha T^{\beta}}{\beta+1}(1+r)+CR(\frac{IT}{2}+r)-\frac{PI_eR}{T}(M_1-\frac{T}{2}) \qquad T<M_1 \quad (3.3.2.20)$$

$$K_3(T)=\frac{A}{T}+\frac{CR\alpha T^{\beta}}{\beta+1}+\frac{CIRT}{2}+\frac{CI_cR}{T}[\ \frac{T^2}{2}-M(T-\frac{M}{2})\ ]$$

$$-\frac{CI_cR\alpha^{\beta+1}}{T(\beta+1)^2}[\ \frac{T^{2(\beta+1)}}{2}-M^{\beta+1}(T^{\beta+1}-\frac{M^{\beta+1}}{2})\ ]-\frac{PI_eRM^2}{2T} \quad (3.3.2.21)$$

$$K_4(T)=\frac{A}{T}+\frac{CR\alpha T^{\beta}}{\beta+1}+\frac{CIRT}{2}-\frac{PI_eR}{T}(M-\frac{T}{2}) \qquad T<M \quad (3.3.2.22)$$

The first order condition for $K_1(T)$ in (3.3.2.19) to be minimized is $\frac{dK_1(T)}{dT}=0$

$$-\frac{A}{T}+\frac{CR\alpha\beta T^{\beta-1}}{\beta+1}(1+r)+\frac{CIR}{2}-\frac{CI_CR}{T^2}[\ \frac{T^2}{2}-M_1(T-\frac{M_1}{2})\ ]$$

$$+\frac{CI_CR}{T}(T-M_1)+\frac{CI_CR\alpha^{\beta+1}}{T^2(\beta+1)^2}[\ \frac{T^{2(\beta+1)}}{2}-M_1^{\beta+1}(T^{\beta+1}-\frac{M_1^{\beta+1}}{2})\ ]$$

$$-\frac{CI_CR\alpha^{\beta+1}}{T(\beta+1)}[\ T^{2\beta+1}-M_1^{\beta+1}T^{\beta}\ ]+\frac{PI_eRM_1^2}{2T^2}=0 \quad (3.3.2.23)$$

For the second order condition, we obtain

$$\frac{d^2K_1(T)}{dT^2}=\frac{2A}{T^3}+\frac{CR\alpha\beta(\beta-1)T^{\beta-2}}{\beta+1}(1+r)+\frac{2CI_cR}{T^3}[\ \frac{T^2}{2}-M_1(T-\frac{M_1}{2})\ ]$$

$$-\frac{CI_cR}{T^2}(T-M_1)-\frac{CI_cR}{T^2}(T-M_1)+\frac{2CI_cR\alpha^{\beta+1}}{T^2(\beta+1)^2}[\ T^{2\beta+1}-M_1^{\beta+1}T^{\beta}\ ]$$

$$-\frac{2CI_cR\alpha^{\beta+1}}{T^3(\beta+1)^2}[\ \frac{T^{2(\beta+1)}}{2}-M_1^{\beta+1}(T^{\beta+1}-\frac{M_1^{\beta+1}}{2})\ ]$$

$$-\frac{CI_cR\alpha^{\beta+1}}{T(\beta+1)}[\ (2\beta+1)T^{2\beta}-\beta M_1^{\beta+1}T^{\beta-1}\ ]-\frac{PI_eRM_1^2}{T^3} \quad (3.3.2.24)$$

Equation (3.3.2.23) is highly non-linear. So it can be solved by using suitable numerical method. (e.g. Newton – Raphson Method). Consequently, we obtain optimal value of $T = T_1$ for case 1. Ensure that $T_1 > M_1$.

The first order condition for case 2 is $\frac{dK_2(T)}{dT} = 0$

$$-\frac{A}{T} + \frac{CR\alpha\beta T^{\beta-1}}{\beta+1}(1+r) + \frac{CIR}{2} - \frac{PI_eR}{T^2}(M_1 - \frac{T}{2}) + \frac{PI_eR}{2T} = 0 \tag{3.3.2.25}$$

and for the second order condition, we obtain

$$\frac{d^2K_2(T)}{dT^2} = \frac{2A}{T^3} + \frac{CR\alpha\beta(\beta-1)T^{\beta-2}}{\beta+1}(1+r) - \frac{2PI_eRM_1}{T^3} \tag{3.3.2.26}$$

Solving (3.3.2.25) by suitable numerical method, we obtain the optimal value of $T=T_2$ for case 2. Ensure that $T_2 < M_1$.

Arguing as above, the first order condition for finding the optimal value of $T = T_3$ for case 3 is $\frac{dK_3(T)}{dT} = 0$

$$\begin{aligned} &-\frac{A}{T^2} + \frac{CR\alpha\beta T^{\beta-1}}{\beta+1} + \frac{CIR}{2} - \frac{CI_cR}{T^2}\left[\frac{T^2}{2} - M(T - \frac{M}{2})\right] \\ &+\frac{CI_cR}{T}(T-M) + \frac{CI_cR\alpha^{\beta+1}}{T^2(\beta+1)^2}\left[\frac{T^{2(\beta+1)}}{2} - M^{\beta+1}(T^{\beta+1} - \frac{M^{\beta+1}}{2})\right] \\ &-\frac{CI_cR\alpha^{\beta+1}}{T(\beta+1)}\left[T^{2\beta+1} - M^{\beta+1}T^{\beta}\right] + \frac{PI_eRM^2}{2T^2} = 0 \end{aligned} \tag{3.3.2.27}$$

and for the second order condition, we obtain

$$\frac{d^2K_3(T)}{dT^2}=\frac{2A}{T^3}+\frac{CR\alpha\beta(\beta-1)T^{\beta-2}}{\beta+1}+\frac{2CI_cR}{T^3}\left[\frac{T^2}{2}-M(T-\frac{M}{2})\right]$$

$$-\frac{CI_cR}{T^2}(T-M)-\frac{CI_cR}{T^2}(T-M)+\frac{CI_cR}{T}+\frac{2CI_cR\alpha^{\beta+1}}{T^2(\beta+1)^2}\left[T^{2\beta+1}-M^{\beta+1}T^{\beta}\right]$$

$$-\frac{2CI_cR\alpha^{\beta+1}}{T^3(\beta+1)^2}\left[\frac{T^{2(\beta+1)}}{2}-M^{\beta+1}(T^{\beta+1}-\frac{M^{\beta+1}}{2})\right]$$

$$-\frac{CI_cR\alpha^{\beta+1}}{T(\beta+1)}\left[(2\beta+1)T^{2\beta}-\beta M^{\beta+1}T^{\beta-1}\right]-\frac{PI_eRM^2}{T^3} \tag{3.3.2.28}$$

For case 4, the first order condition is

$$-\frac{A}{T^2}+\frac{CR\alpha\beta T^{\beta-1}}{\beta+1}+\frac{CIR}{2}+\frac{PI_eR}{T^2}M=0 \tag{3.3.2.29}$$

and for the second order condition, we obtain

$$\frac{d^2K_4(T)}{dT^2}=\frac{2A}{T^3}+\frac{CR\alpha\beta(\beta-1)T^{\beta-2}}{\beta+1}-\frac{2pI_eRM}{T^3} \tag{3.3.2.30}$$

Eq. (3.3.2.29) gives optimum value of $T=T_4$. Ensure that $T_4<M$.

3.3.3 Numerical Example and observations :

Consider numerical value of parameters in proper units :

$$[A, C, I, I_e, r]=[250, 20, 10\%, 12\%, 2\%]$$

The following tables give effect of various parameters on optimum cycle time, optimum purchase quantity and total relevant cost per year.

Table – 3.3.3.1

$\alpha\backslash R$		**1000**	**500**	**750**
0.01	T	0.3116	0.4399	0.3596
	Q	311.89	220.20	269.95
	$K_1(T)$	1870.57	1267.86	1589.13
0.02	T	0.3076	0.4332	0.3546
	Q	308.08	217.11	266.45
	$K_1(T)$	1884.63	1279.63	1602.19
0.03	T	0.3038	0.4269	0.3499
	Q	304.45	214.19	263.13
	$K_1(T)$	1898.43	1291.13	1614.99

Increase in the demand rate reduces cycle time, increases procurement quantity and total cost of the inventory system while increase in the deterioration rate decreases optimal procurement quantity and increases the total cost of the inventory system significantly.

Table – 3.3.3.2

$\alpha\backslash\beta$		**1.5**	**2**	**2.5**
0.01	T	0.3116	0.3132	0.3142
	Q	311.89	313.35	314.34
	$K_1(T)$	1870.57	1862.96	1859.48
0.02	T	0.3076	0.3106	0.3127
	Q	308.08	310.89	312.83
	$K_1(T)$	1884.63	1869.58	1862.69
0.03	T	0.3038	0.3082	0.3112
	Q	304.45	308.54	311.36
	$K_1(T)$	1898.43	1876.09	1865.86

Keeping α fixed, increase in shape parameter β increases cycle time and procurement quantity while decreases the total cost of the inventory system.

Table – 3.3.3.3

$\beta \backslash R$		500	750	1000
1.5	T	0.4399	0.3596	0.3116
	Q	220.20	269.95	311.89
	$K_1(T)$	1267.86	1589.13	1870.57
2	T	0.4417	0.3613	0.3132
	Q	220.99	271.12	313.35
	$K_1(T)$	1262.52	1582.51	1862.96
2.5	T	0.4431	0.3625	0.3142
	Q	221.67	271.98	314.34
	$K_1(T)$	1259.66	1579.27	1859.48

Increase in demand rate decreases the cycle time, increases the optimal procurement quantity and significantly increases the total cost of the inventory system. The increase in β decreases the total cost of an inventory system and increases the optimal procurement quantity.

Table – 3.3.3.4

$R \backslash P$		30	40	50
500	T	0.4399	0.4394	0.4390
	Q	220.20	219.98	219.76
	$K_1(T)$	1267.86	1266.71	1265.55
750	T	0.3596	0.3590	0.3585
	Q	269.95	269.54	269.13
	$K_1(T)$	1589.13	1587.01	1584.89
1000	T	0.3116	0.3110	0.3104
	Q	311.89	311.26	310.63
	$K_1(T)$	1870.57	1867.32	1864.06

Increase in the selling price decreases the number of units to be procured and as a result total cost of an inventory system decreases. For fixed selling price increase in demand increases the cost of the inventory system and the procurement quantity significantly.

Table – 3.3.3.5

$\alpha \backslash P$		30	40	50
0.01	T	0.3116	0.3110	0.3104
	Q	311.89	311.26	310.63
	$K_1(T)$	1870.57	1867.32	1864.06
0.02	T	0.3076	0.3070	0.3064
	Q	308.08	307.46	306.84
	$K_1(T)$	1884.63	1881.34	1878.03
0.03	T	0.3068	0.3032	0.3026
	Q	304.45	303.84	303.23
	$K_1(T)$	1898.43	1895.09	1891.74

For fixed α , increase in the selling price decreases the values of the decision variables. For fixed selling price, increase in deterioration rate α , reduces the number of units to be purchased but increases total cost of the inventory system.

Table –3.3.3.6

$\beta \backslash P$		30	40	50
1.5	T	0.3116	0.3110	0.3104
	Q	311.89	311.26	310.63
	$K_1(T)$	1870.57	1867.32	1864.06
2	T	0.3132	0.3126	0.3119
	Q	313.35	312.71	312.08
	$K_1(T)$	1862.96	1857.72	1856.48
2.5	T	0.3142	0.3136	0.3130
	Q	314.34	313.71	313.07
	$K_1(T)$	1859.48	1856.25	1853.02

For fixed shape parameter β, increase in the selling price reduces optimum quantity to be purchased and total cost of the inventory system. For fixed selling price, increase in the shape parameter β decreases the total cost of an inventory system and increases the number of units to be purchased.

Table – 3.3.3.7

$\alpha \backslash M$		15 / 365	30 / 365	45 / 365
0.01	T	0.3116	0.3107	0.3091
	Q	311.89	310.94	309.36
	$K_1(T)$	1870.57	1742.39	1610.91
0.02	T	0.3076	0.3067	0.3051
	Q	308.08	307.15	305.59
	$K_1(T)$	1884.63	1756.38	1624.80
0.03	T	0.3068	0.3029	0.3014
	Q	304.45	303.54	302.01
	$K_1(T)$	1898.43	1770.12	1638.43

Increase in the delay period decreases the number of units to be purchased and the total cost of the inventory system. For fixed allowable credit period, increase in α, reduces the number of units to be purchased and increases the total cost of an inventory system.

Table – 3.3.3.8

$\beta \backslash M$		15 / 365	30 / 365	45 / 365
1.5	T	0.3116	0.3107	0.3091
	Q	311.89	310.94	309.36
	$K_1(T)$	1870.57	1734.80	1610.91
2.0	T	0.3132	0.3123	0.3107
	Q	313.35	312.40	310.81
	$K_1(T)$	1862.96	1734.80	1603.36
2.5	T	0.3142	0.3133	0.3117
	Q	314.34	313.39	311.80
	$K_1(T)$	1859.48	1731.33	1599.93

Increase in the delay period for fixed shape parameter, decreases the number of units to be procured and the total cost of the inventory system. For fixed allowable credit period, increase in the shape parameter increases the number of units to be purchased and decreases the total cost of the inventory system.

Table – 3.3.3.9

$P \setminus M$		15 / 365	30 / 365	45 / 365
30	T	0.3116	0.3107	0.3091
	Q	311.89	310.94	309.36
	$K_1(T)$	1870.57	1742.39	1610.91
40	T	0.3110	0.3081	0.3033
	Q	311.26	308.40	303.59
	$K_1(T)$	1867.32	1729.29	1581.13
50	T	0.3104	0.3056	0.2975
	Q	310.63	305.84	297.70
	$K_1(T)$	1864.06	1716.08	1550.77

For fixed allowable credit period, increase in the selling price reduces the number of units to be procured and the total cost of the inventory system. For fixed selling price, increase in the allowable credit period reduces the number of units procured and the total cost of the inventory system.

Table – 3.3.3.10

$I_c \setminus M$		15 / 365	30 / 365	45 / 365
0.12	T	0.3311	0.3291	0.3257
	Q	331.40	329.39	326.00
	$K_1(T)$	1797.32	1689.47	1575.35
0.15	T	0.3116	0.3107	0.3091
	Q	311.89	310.94	309.36
	$K_1(T)$	1870.57	1742.39	1610.91
0.18	T	0.2953	0.2953	0.2953
	Q	295.50	295.50	295.50
	$K_1(T)$	1938.57	1790.61	1642.65

For fixed delay period, increase in the interest charges to be paid reduces the cycle time and number of units to be procured and increases the total cost of the inventory system.

3.4 Conclusion :

In this chapter three mathematical models are formulated. Section 3.1 is a lot-size model with time dependent deterioration of units and permissible delay in payments which is extended by allowing shortages to an order level lot size model in Section 3.2. Different scenarios of permissible delay in payments are discussed. It is found that the model in Section 3.1 is sensitive to the rate of deterioration of units in the inventory system, demand rate, purchase price of a unit, ordering cost and the allowable delay period. The model in Section 3.2 is also sensitive to the backorder cost. By putting $T_1 = 0$ in the model in Section 3.2, it reduces to the model in Section 3.1. These models can be extended by introducing inflation rate, price dependent demand etc.

In Section 3.3 an attempt is made to develop an EOQ model for time dependent deteriorating items to determine the optimal ordering policy when the supplier offers cash discount and a permissible delay in payments. The Taylor series approximation is used to derive analytic results. A numerical example is provided to verify the results obtained in market. The proposed model can be extended taking demand to be a function of selling price, time varying and stock dependent. It can be generalized to allow for shortages and inflation rates.

CHAPTER-4

AN EOQ MODEL FOR DETERIORATING ITEMS WITH PRICE DEPENDENT DEMAND UNDER SUPPLIER CREDITS

4.0 Introduction :

In this paper, an EOQ model for constant rate of deteriorating items and selling price dependent demand, in which the supplier provides not only a cash discount but also a credit period to the customer, is discussed. The characterization of the optimal solution and an easy-to-use algorithm is given to find optimum selling price, optimal ordering quantity, and optimum replenishment time to maximize the net profit. Finally, the numerical example is given to see the interdependence of inventory parameters on decision variables and hence objective function.

4.1 Assumptions and Notations.

The following assumptions and notations other than A.1 and N.1 are used to develop the proposed mathematical model.

1. The demand is $R(p)= a-bp$, $(a, b > 0,\ a >> b)$, p denotes selling price of the item.
2. Shortages are not allowed.
3. Time horizon is infinite.

The following notations are used throughout the paper

C = the unit purchasing cost, with $C < p$

r = the cash discount rate, $0 < r < 1$

θ = the constant deterioration rate, $0 \leq \theta < 1$

M_1 = the period of cash discount

M = the period of permissible delay in settling account, with $M > M_1$

p, Q and T are decision variables

$K(p,T)$ = the total relevant cost per year

$NP(p,T)$ = the net profit per year

The total relevant cost consists of (a) Ordering cost, (b) Cost of deteriorated units, (c) Cost of carrying inventory (excluding interest charges), (d) cash discount earned if the payment is made at M_1 (e) Cost of interest charged for unsold items after the permissible delay M, and (f) interest earned from the sales revenue during the permissible period.

4.2 Mathematical Formulation :

The inventory level $I(t)$ gradually decreases partly to meet the demands and partly due to deterioration. The differential equation governing the variation of inventory with respect to time can be given by

$$\frac{dI(t)}{dt} + \theta I(t) = -R(p) \qquad 0 \leqslant t \leqslant T \tag{4.2.1}$$

with the boundary condition $I(0) = Q$ and $I(T) = 0$.

Then the solution of (1) is given by

$$I(t) = \frac{(a-bp)}{\theta}\{e^{\theta(T-1)} - 1\} \tag{4.2.2}$$

And the order quantity is

$$Q = I(0) = \frac{(a-bp)}{\theta}\{e^{\theta T} - 1\} \tag{4.2.3}$$

Total demand during one cycle is $R(p)T$. Hence, the number of deteriorating items during a cycle time is

$$= Q - R(p)\,T$$

$$= \frac{(a-bp)}{\theta}(e^{\theta T} - \theta T - 1) \qquad (4.2.4)$$

The total relevant cost $K(p, T)$ per time unit consists of the following costs.

a) Ordering cost per time unit $= \frac{A}{T}$ (4.2.5)

b) Cost of deteriorated units per time unit $= \frac{C(a-bp)}{\theta T}(e^{\theta T} - \theta T - 1)$ (4.2.6)

c) Cost of carrying inventory per time unit

$$= \frac{h}{T}\int_0^T I(t)dt = \frac{h(a-bp)}{\theta^2 T}(e^{\theta T} - \theta T - 1) \qquad (4.2.7)$$

Regarding cash discount, interest charged and earned, there are four possibilities based on the customer's two choices either to pay at M_1 or M and the length of T.

Possibility (1): The payment is made at M_1 to get a cash discount and $T \geq M_1$

Possibility (2): The customer pays in full at M_1 to get a cash discount but $T < M_1$

Possibility (3): The payment is made at time M to get the permissible delay and $T \geq M$

Possibility (4): The customer pays in full at M but $T < M$

Next, we discuss cash discount, the cost of interest charged and interest earned for each of the above mentioned four possibilities.

Possibility (1) : $T \geq M_1$

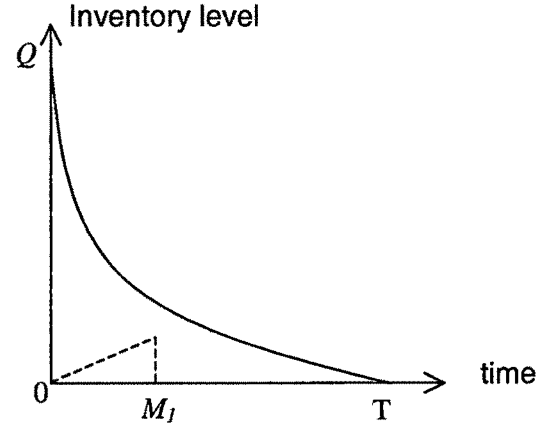


Fig. 4.2.1

Since the payment is made at time M_1, the cash discount per year is given by

$$\frac{rCQ}{T} = \frac{rC(a-bp)}{\theta T}(e^{\theta T} - 1) \tag{4.2.8}$$

The interest charged per year is

$$= \frac{CI_c}{T}\int_{M_1}^{T} I(t)dt = \frac{CI_c(a-bp)}{\theta^2 T}(e^{\theta(T-M_1)} - \theta(T-M_1) - 1) \tag{4.2.9}$$

The interest earned per year is

$$= \frac{pI_e}{T}\int_{0}^{M_1} R(p)tdt$$

$$= \frac{pI_e(a-bp)M_1^2}{2T} \tag{4.2.10}$$

Using (4.2.5) – (4.2.10), the net profit per year $NP_1(p, T)$ is

$$NP_1(p,T) = (p-C)(a-bp) - \frac{A}{T} - \frac{C(a-bp)}{\theta T}(e^{\theta T} - \theta T - 1)$$

$$- \frac{h(a-bp)}{\theta^2 T}(e^{\theta T} - \theta T - 1) - \frac{rC(a-bp)}{\theta T}(e^{\theta T} - 1)$$

$$- \frac{CI_c(a-bp)}{\theta^2 T}(e^{\theta(T-M_1)} - \theta(T-M_1) - 1) + \frac{pI_c(a-bp)M_1^2}{2T} \tag{4.2.11}$$

Possibility (2) : $T < M_1$

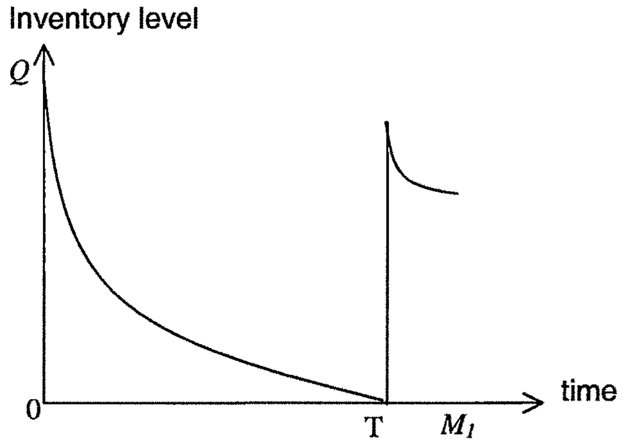


Fig. 4.2.2

Here, there is no interest charged, cash discount is the same as that in possibility(1).

The interest earned per year is

$$= \frac{pI_e}{T}\left[\int_0^T R(p)t\,dt + R(p)T(M_1 - T) \right]$$

$$= pI_e(a-bp)(M_1 - \frac{T}{2}) \qquad (4.2.12)$$

Hence, net profit $NP_2\,(p, T)$ per year is

$$NP_2(p,T) = (p-C)(a-bp) - \frac{A}{T} - \frac{C(a-bp)}{\theta T}(e^{\theta T} - \theta T - 1)$$

$$- \frac{h(a-bp)}{\theta^2 T}(e^{\theta T} - \theta T - 1) - \frac{rC(a-bp)}{\theta T}(e^{\theta T} - 1)$$

$$- pI_e(a-bp)(M_1 - \frac{T}{2}) \qquad (4.2.13)$$

Possibility (3) : $T \geq M$

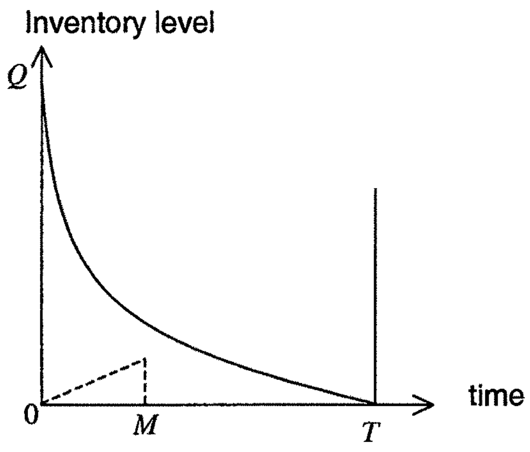


Fig. 4.2.3

Since the payment is made at time *M*, there is no cash discount. The interest charged per year is

$$= \frac{CI_c}{T} \int_M^T I(t)dt = \frac{CI_c(a-bp)}{\theta^2 T}(e^{\theta(T-M)} - \theta(T-M) - 1) \quad (4.2.14)$$

The interest earned per year is

$$\frac{pI_e}{T} \int_0^M R(p)tdt = \frac{pI_e(a-bp)M^2}{2T} \quad (4.2.15)$$

Therefore, the net profit $NP_3(p, T)$ is

$$NP_3(p,T) = (p-C)(a-bp) - \frac{A}{T} - \frac{C(a-bp)}{\theta T}(e^{\theta T} - \theta T - 1)$$

$$- \frac{h(a-bp)}{\theta^2 T}(e^{\theta T} - \theta T - 1) - \frac{CI_c(a-bp)}{\theta^2 T}(e^{\theta(T-M)} - \theta(T-M) - 1)$$

$$+ \frac{pI_e(a-bp)M^2}{2T} \quad (4.2.16)$$

Possibility (4) : $T < M$

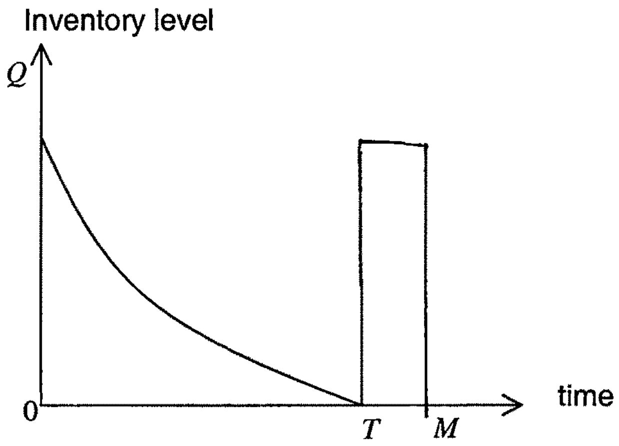


Fig. 3.2.4

In this case, there is no interest charged.

The interest earned per year is

$$= \frac{pI_e}{T}\left[\int_0^T R(p)t\,dt + R(p)T(M-T) \right]$$

$$= pI_e(a-bp)(M-\frac{T}{2}) \qquad (4.2.17)$$

Hence, the net profit $NP_4(p,T)$ per year is

$$NP_4(p,T) = (p-C)(a-bp) - \frac{A}{T} - \frac{C(a-bp)}{\theta T}(e^{\theta T} - \theta T - 1)$$

$$- \frac{h(a-bp)}{\theta^2 T}(e^{\theta T} - \theta T - 1) + pI_e(a-bp)(M-\frac{T}{2}) \qquad (4.2.18)$$

Theoretical Results:

The first order condition for $NP_1(p,T)$ in (4.2.11) to be maximum is

$$\frac{\partial NP_1(p,T)}{\partial p} = 0 \text{ and } \frac{\partial NP_1(p,T)}{\partial T} = 0$$

$$\frac{\partial NP_1(p,T)}{\partial p} = 0 \text{ gives}$$

$$p = \frac{a}{2b} + \frac{1}{b(2T + I_e M_1^2)}\Big[\frac{CI_c b}{\theta^2 T}(e^{\theta(T-M_1)} - \theta(T-M_1) - 1) + b\{h + C\theta(1-r)\}(e^{\theta T} - 1) - \frac{hb}{\theta} \Big] \tag{4.2.19}$$

Substituting this in (4.2.11) and taking Taylor's Series approximation of $e^{\theta T}$ by neglecting higher powers of θ and r, we get,

$$NP_1(T) = -\frac{A}{T} + \Big[\frac{a}{2} - \frac{b}{(2T + I_e M_1^2)}\{CI_c(\frac{(T-M_1)^2}{2} + \frac{\theta(T-M_1)^3}{6}) + h\theta T^2 - \frac{hT}{\theta}\} \Big] * \Big[\frac{-CI_c}{2T}(\frac{(T-M_1)^2}{2} + \frac{\theta(T-M_1)^3}{6}) + \frac{h\theta T}{2} - \frac{h}{2\theta} - \frac{hT}{2} - \frac{h\theta T^2}{6} + \frac{a}{2b}(1 + \frac{I_e M_1^2}{2T}) - C(1-r)(1 + \frac{\theta T}{2}) \Big] \tag{4.2.20}$$

And hence, $\frac{\partial NP_1(T)}{\partial T} = 0$ can be solved by Newton – Raphson's method.

Call this solution to be T_1.

Arguing in a similar way for possibility (2), $\frac{\partial NP_2(p,T)}{\partial p} = 0$ giving us

$$p=\frac{a}{2b}+\frac{1}{b(1-I_e(M_1-\frac{T}{2})}\left[\frac{bc(e^{\theta T}-\theta T-1)}{\theta T}+\frac{bh(e^{\theta T}-\theta T-1)}{\theta^2 T}+\frac{brc(e^{\theta T}-1)}{\theta T}+cb\right] \quad (4.2.21)$$

and hence,

$$NP_2(T)=-\frac{A}{T}+\left[a-b\{\frac{a}{2b}+\frac{1}{1-I_e(M_1-T/2)}\{c(1+r)(1+\frac{\theta T}{2})+\frac{hT}{2}(1+\frac{\theta T}{3})\}\}\right]$$
$$*\left[(1-I_e(M_1-T/2))\{\frac{a}{2b}+\frac{1}{1-I_e(M_1-T/2)}\{c(1+r)(1+\frac{\theta T}{2})+\frac{hT}{2}(1+\frac{\theta T}{3})\}\}\right.$$
$$\left.-C-\frac{C\theta T}{2}-\frac{hT}{2}(1+\frac{\theta T}{3})-rC(1+\frac{\theta}{2})\right] \quad (4.2.22)$$

Similarly, $\frac{\partial NP_3(p,T)}{\partial p} = 0$ from (4.2.16) gives

$$p=\frac{a}{2b}+\frac{T}{b(2T+I_eM^2)}\left[bC+\frac{bC(e^{\theta T}-\theta T-1)}{\theta T}+\frac{bh(e^{\theta T}-\theta T-1)}{\theta^2 T}+\frac{CI_cb}{\theta^2 T}(e^{\theta(T-M)}-\theta(T-M)-1)\right] \quad (4.2.23)$$

and hence,

$$NP_3(T)=-\frac{A}{T}+\left[a-b\{\frac{a}{2b}+\frac{T}{2T+I_eM^2}\{bc(1+\frac{\theta T}{2})+\frac{hb}{2}(T+\frac{\theta T^2}{3})\right.$$
$$\left.+\frac{CI_cb}{2T}((T-M)^2+\theta\frac{(T-M)^3}{6}\}\}\right]*\left[(1+\frac{I_eM^2}{2T})\{\frac{a}{2b}+\frac{T}{2T+I_eM^2}\{bc(1+\frac{\theta T}{2})\right.$$
$$+\frac{hb}{2}(T+\frac{\theta T^2}{3})+\frac{CI_cb}{2T}((T-M)^2+\theta\frac{(T-M)^3}{6}\}\}-C-\frac{C(1+\theta T)}{2}$$
$$\left.+h\{\frac{1}{\theta}+\frac{T}{2}+\frac{\theta T^2}{6}-\frac{CI_c}{2T}((T-M)^2+\frac{\theta(T-M)^3}{3}\}\right] \quad (4.2.24)$$

Similarly $\frac{\partial NP_4(p,T)}{\partial p} = 0$ for (4.2.18) gives

$$p = \frac{a}{2b} + \frac{1}{b(2-I_e(M-T))}\left[\frac{bC(e^{\theta T} - \theta T - 1)}{\theta T} + \frac{bh(e^{\theta T} - \theta T - 1)}{\theta^2 T} + Cb \right] \quad (4.2.25)$$

and hence,

$$NP_4(T) = -\frac{A}{T} + \Big[a - b\{\frac{a}{2b} + \frac{1}{b(2-I_e(M-T))}\{hb(T+\frac{\theta T}{3}) + bc(1+\frac{\theta T}{2})\}\}\Big]$$

$$*\Big[(1-I_e(M_1 - T/2))\{\frac{a}{2b} + \frac{1}{b(2-I_e(M-T))}\{\frac{hb}{2}(T+\frac{\theta T}{3}) + bc(1+\frac{\theta T}{2})\}\}\Big]$$

$$-C(1+\frac{\theta T}{2}+\frac{r\theta T}{2}) - \frac{h}{2}(T+\frac{\theta T^2}{3}) \Big] \quad (4.2.26)$$

4.3 Special Cases :

1) As a special case, it we put $M_1 = M$ and $r = \theta = 0$, the model reduces to that of Chung (1998) and Teng (2001).

2) The classical EOQ model is a special case of possibility (1) with $M_1 = 0$ and $r = 0$ (or possibility (3) with $M = 0$)

4.4 Numerical Example and observations :

Consider the inventory system with

K = \$ 250 / order, a = 2000, b = 12, h = \$ 0.2 / unit / year, C = \$ 20 per unit, I_c = 15% / unit / year , I_e = 12% / unit / year, M_1 = 15 days = 15 / 365 years, M = 30 days = 30 / 365 year

Table

Variable		T	Q	p	NP
a	1000	0.1445	80.95	36.72	12217.98
	2000	0.1236	128.73	80.04	21414.23
	3000	0.0964	148.66	121.69	31714.57
b	10	0.1209	125.05	96.71	21480.12
	12	0.1236	128.73	80.04	21414.23
	14	0.1266	132.74	68.14	21350.44
A	150	0.0951	99.04	80.02	20984.45
	200	0.1101	114.94	80.03	21216.52
	250	0.1236	128.76	80.04	21414.33
M_1	15/365	0.1236	128.76	80.04	21414.33
	30/365	0.1258	131.19	80.01	21415.57
	45/365	0.1308	136.31	79.99	21355.00
I_e	0.11	0.1235	128.69	80.04	21415.92
	0.12	0.1236	128.76	80.04	21414.33
	0.13	0.1237	128.90	80.04	21413.00
I_c	0.14	0.1235	128.74	80.03	21418.41
	0.15	0.1236	128.74	80.04	21414.26
	0.16	0.1236	128.74	80.04	21410.16
R	0.2	0.1236	128.74	80.04	21414.26
	0.3	0.1218	126.90	80.04	19443.45
	0.4	0.1202	125.14	80.04	17472.20
θ	0.02	0.1502	158.52	78.85	24087.39
	0.03	0.1236	128.76	80.04	21414.33
	0.04	0.1042	108.48	80.09	20087.39

Observations:

- Increase in a, the constant demand factor reduces optimum cycle time, increases optimum purchase quantity, selling price and net profit of the system. The decision variables and the objective function are very sensitive to this factor.

- Increase in b, results in increase in optimum cycle time and optimum procurement quantity but significant decrease in optimal selling price and net profit of the system.
- Increase in ordering cost increases optimum cycle time and purchase quantity and decreases net profit. Selling price of an item is insensitive to changes in ordering cost.
- Optimum cycle time, purchase quantity and net profit increases with increase in the credit period. Here selling price is again insensitive to allowable delay period.
- Increase in interest earned increases cycle time, purchase quantity and net profit in very little margin.
- Increase in interest charge reduces net profit of the system.
- Increase in discount rate decreases optimum cycle time and purchase quantity. Net profit is very sensitive to changes in cash discount factor, r.
- Increase in deterioration rate of items reduces optimum cycle time, purchase quantity and net profit significantly.

4.5 Conclusions :

In this chapter, an EOQ model for deteriorating items with price dependent demand is developed to determine the optimal ordering policy when the supplier provides a cash discount and / or a permissible delay in payment. Taylor's series approximation is used to obtain the solution. The effects of changes in parametric values are studied on the decision variables and objective function. The proposed model can be extended to allow for shortages, quantity discounts and inflation rates.

CHAPTER 5

AN EOQ MODEL FOR DETERIORATING ITEMS WITH SELLING PRICE AND STOCK DEPENDENT DEMAND DURING INFLATION UNDER SUPPLIER CREDITS.

5.0 Introduction :

In this chapter, an inventory model is developed for deteriorating items for which the demand is dependent on the selling price as well as the stock. The units in inventory are subject to constant rate of deterioration. Shortages are not allowed and the supplier provides a cash discount and a permissible delay in payments. The optimal solution is characterized to optimize the net profit. An easy-to-use algorithm is given to find the optimal selling price, the optimal order quantity and replenishment cycle time that maximizes the net profit. At the end, a numerical example is given to illustrate the theoretical results and sensitivity analysis of parameters on the optimal solutions is carried out.

5.1 Notations and Assumptions:

The following additional notations and assumptions other than N.1 and A.1 given earlier are used in the chapter.

Assumptions:

1 Shortages are not allowed.

2. The demand rate function $R(t)$ is deterministic selling price dependent and is a known function of instantaneous stock level $I(t)$. Take functional form of $R(t)$ as

$$R(p,I(t)) = \alpha - \beta p + \delta I(t), \qquad 0 \le t \le T \qquad (5.2.1)$$

where $\alpha > 0,\ \alpha >> \beta,\ 0 < \delta < 1$

Notations:

θ =The constant deterioration rate

r =Cash discount rate

M =Permissible delay in settling the accounts and

$R(p,I(t))$ =The demand dependent on the selling price as well as the stock per unit time

$$R(p,I(t)) = \alpha - \beta p + \delta I(t),$$

where α, is the fixed demand and $\alpha > 0$

β and δ constants, $\alpha >> \beta$, $\alpha >> \delta$

p and Q are the decision variables.

5.2 Mathematical Formulation :

The depletion of inventory occurs due to the combined effects of the demand and deterioration of units in the interval $[0, T]$. Hence, the variation of inventory level $I(t)$, with respect to time can be governed by the differential equation

$$\frac{dI(t)}{dt} + \theta I(t) = -(\alpha - \beta p + \delta I(t)), \qquad 0 \le t \le T \qquad (5.2.2)$$

with the boundary condition $I(0) = Q$ and $I(T) = 0$, the solution of (5.2.2) is given by

$$I(t) = \frac{(\alpha - \beta p)}{(\theta + \delta)}[e^{(\theta+\delta)(T-t)} - 1], \qquad 0 \le t \le T \qquad (5.2.3)$$

and the order quantity is

$$Q = I(0) = \frac{(\alpha - \beta p)}{(\theta + \delta)}[e^{(\theta+\delta)T} - 1] \qquad (5.2.4)$$

From (5.2.3), the inventory holding cost, IHC, in the interval $[0,T]$, per time unit is

$$IHC = \frac{h}{T}\int_0^T I(t)dt$$

$$= \frac{h}{T}\frac{(\alpha - \beta p)}{(\theta + \delta)^2}[e^{(\theta+\delta)T} - (\theta+\delta)T - 1] \qquad (5.2.5)$$

and the number of units deteriorated during interval $[0,T]$, is

$$I(0) - \int_0^T (\alpha - \beta p + \delta I(t))dt$$

Then the deterioration cost, CD, per time unit is

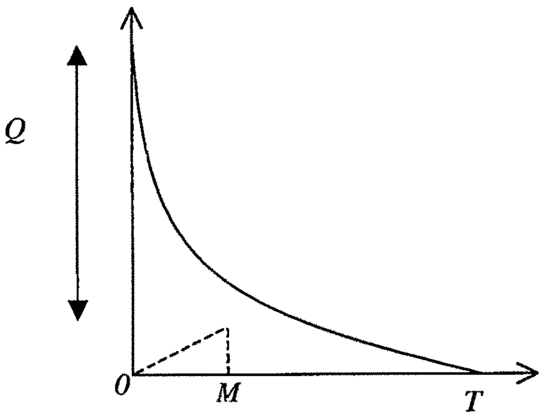
$$CD = \frac{C(\alpha - \beta p)}{T(\theta + \delta)^2}[(e^{(\theta+\delta)T} - 1)(\theta+\delta) - T(\theta+\delta) - \delta(e^{(\theta+\delta)T} - (\theta+\delta)T - 1)] \qquad (5.2.6)$$

The ordering cost, OC, per time unit is

$$OC = \frac{A}{T} \tag{5.2.7}$$

Now, regarding the period M of permissible delay in settlement of accounts, there are two possibilities: Case (1): $M \le T$ and Case (2): $M > T$

Case (1) : $M \le T$



Here, the length of the period is greater than the credit period. So, the buyer can use the sales revenue to earn the interest with an annual rate I_e in $[0,M]$, denoted by IE_1. Then,

$$IE_1 = \frac{pI_e}{T}\int_0^M (\alpha-\beta p+\delta I(t))t\,dt$$

$$= \frac{pI_e}{T}(\alpha-\beta p)\left[\frac{M^2}{2} - \frac{\delta}{(\theta+\delta)^3}\{Me^{(\theta+\delta)(T-M)}(\theta+\delta)+e^{(\theta+\delta)(T-M)}-1+\frac{M^2}{2}(\theta+\delta)^2\}\right] \tag{5.2.8}$$

Here the payment is paid to be made at time M where $M \le T$.

The cash discount CDC per year is given by

$$CDC = \frac{rCQ}{T} = \frac{rC}{T}\frac{(\alpha-\beta p)}{(\theta+\delta)}[e^{(\theta+\delta)T}-1] \tag{5.2.9}$$

The interest charged IC_1 per year is

$$IC_1 = \frac{CI_c}{T}\int_M^T I(t)dt$$

$$= \frac{CI_c}{T}\frac{(\alpha-\beta p)}{(\theta+\delta)^2}[e^{(\theta+\delta)(T-M)}-(\theta+\delta)(T-M)-1] \tag{5.2.10}$$

Now the net profit NP_1 using equation (5.2.5) – (5.2.10) is

$$NP_1(p,T) = (p-C)R(p,I(0)) - CD - IHC - OC - CDC + IE_1 - IC_1 \quad (5.2.11)$$

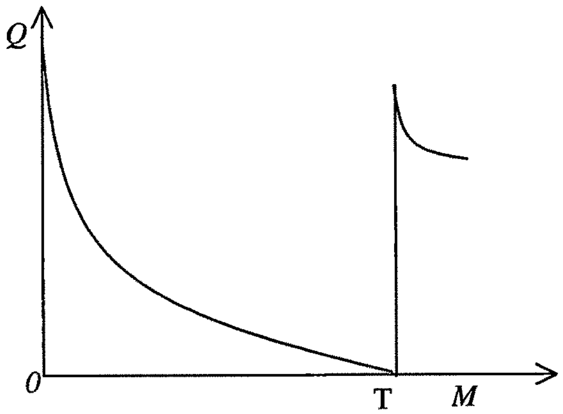
$$= (p-C)R(p,I(0)) - \frac{C(\alpha-\beta p)}{T(\theta+\delta)^2}[(e^{(\theta+\delta)T}-1)(\theta+\delta) - T(\theta+\delta) - \delta(e^{(\theta+\delta)T} - (\theta+\delta)T - 1)]$$

$$-\frac{h}{T}\frac{(\alpha-\beta p)}{(\theta+\delta)^2}[e^{(\theta+\delta)T} - (\theta+\delta)T - 1] - \frac{A}{T} - \frac{rC}{T}\frac{(\alpha-\beta p)}{(\theta+\delta)}[e^{(\theta+\delta)T} - 1]$$

$$+\frac{pI_e}{T}(\alpha-\beta p)[\frac{M^2}{2} - \frac{\delta}{(\theta+\delta)^3}\{Me^{(\theta+\delta)(T-M)}(\theta+\delta) + e^{(\theta+\delta)(T-M)} - 1 + \frac{M^2}{2}(\theta+\delta)^2\}]$$

$$-\frac{CI_c}{T}\frac{(\alpha-\beta p)}{(\theta+\delta)^2}[e^{(\theta+\delta)(T-M)} - (\theta+\delta)(T-M) - 1] \quad (5.2.12)$$

Case (2) : $M > T$



Here there is no interest charged.

The cash discount is the same as in case (1).

The interest earned per year IE_2

$$IE_2 = \frac{pI_e}{T}[\int_0^T R(p,I(t)tdt + R(p,I(0))(M-T)]$$

$$= \frac{pI_e}{T}(\alpha-\beta p)[\frac{\delta}{(\theta+\delta)^3} + \frac{\delta T}{(\theta+\delta)^2} + \frac{\theta T^2}{2(\theta+\delta)} + \{1 + \frac{\delta(e^{(\theta+\delta)T}-1)}{(\theta+\delta)}\}(M-T)] \quad (5.2.13)$$

Thus net profit for case(2) is

$$NP_2(p,T) = (p-C)R(p,I(0)) - CD - IHC - OC - CDC + IE_2 \tag{5.2.14}$$

$$= (p-C)R(p,I(0)) - \frac{C(\alpha-\beta p)}{T(\theta+\delta)^2}[(e^{(\theta+\delta)T}-1)(\theta+\delta) - T(\theta+\delta) - \delta(e^{(\theta+\delta)T}-(\theta+\delta)T-1)]$$

$$-\frac{h}{T}\frac{(\alpha-\beta p)}{(\theta+\delta)^2}[e^{(\theta+\delta)T}-(\theta+\delta)T-1] - \frac{A}{T} - \frac{rC}{T}\frac{(\alpha-\beta p)}{(\theta+\delta)}[e^{(\theta+\delta)T}-1]$$

$$+\frac{pI_e}{T}(\alpha-\beta p)[\frac{\delta}{(\theta+\delta)^3} + \frac{\delta T}{(\theta+\delta)^2} + \frac{\theta T^2}{2(\theta+\delta)} + \{1+\frac{\delta(e^{(\theta+\delta)T}-1)}{(\theta+\delta)}\}(M-T)] \tag{5.2.15}$$

Theoretical Results:

The first order condition for $NP_1(p,T)$ in (5.2.12) to be maximum is

$\frac{\partial NP_1(P,T)}{\partial P} = 0$ and $\frac{\partial NP_1(P,T)}{\partial T} = 0$

$\frac{\partial NP_1(P,T)}{\partial P} = 0$ gives

$$p = \frac{\%X}{\%Y}$$

where

$$\%X = (\alpha + C\beta)(1+\frac{\delta(e^{(\theta+\delta)T}-1)}{\theta+\delta}) + \beta\{\frac{C(e^{(\theta+\delta)T}-1-T-\frac{\delta(e^{(\theta+\delta)T}-(\theta+\delta)T-1)}{(\theta+\delta)})}{(\theta+\delta)T}$$

$$+\frac{h(e^{(\theta+\delta)T}-(\theta+\delta)T-1)}{(\theta+\delta)^2 T} + \frac{rC(e^{(\theta+\delta)T}-1)}{(\theta+\delta)T} + \frac{CI_c(e^{(\theta+\delta)(T-M)}-(\theta+\delta)(T-M)-1)}{(\theta+\delta)^2 T}\}$$

$$+\frac{I_e\alpha\{0.5M^2 - \frac{\delta(Me^{(\theta+\delta)(T-M)}(\theta+\delta)+e^{(\theta+\delta)(T-M)}-1+0.5M^2(\theta+\delta)^2)}{(\theta+\delta)^3}\}}{T}]$$

$$\%Y = 2\beta[\frac{1+\delta(e^{(\theta+\delta)T}-1)}{(\theta+\delta)} + \frac{I_e\{0.5M^2 - \frac{\delta(Me^{(\theta+\delta)(T-M)}(\theta+\delta)+e^{(\theta+\delta)(T-M)}-1+0.5M^2(\theta+\delta)^2)}{(\theta+\delta)^3}\}}{T}]$$

Substituting this p in (5.2.12), we get $NP_1(T)$

$$NP_1(T)=(\frac{\%7}{\%3}-C)(\alpha-\frac{\beta\%7}{\%3})\%2-\frac{A}{T}-\frac{C(\alpha-\frac{\beta\%7}{\%3})\%6}{(\theta+\delta)T}-\frac{h(\alpha-\frac{\beta\%7}{\%3})\%5}{(\theta+\delta)^2T}$$
$$-\frac{rC(\alpha-\frac{\beta\%7}{\%3})(e^{(\theta+\delta)T}-1)}{(\theta+\delta)T}-\frac{CI_c(\alpha-\frac{\beta\%7}{\%3})\%4}{(\theta+\delta)^2T}+\frac{\%7I_e(\alpha-\frac{\beta\%7}{\%3})\%1}{\%3T}$$

where

$$\%1=0.5M^2-\frac{\delta(Me^{(\theta+\delta)(T-M)}(\theta+\delta)+e^{(\theta+\delta)(T-M)}-1+0.5M^2(\theta+\delta)^2)}{(\theta+\delta)^3}$$

$$\%2=1+\frac{\delta(e^{(\theta+\delta)T}-1)}{\theta+\delta}$$

$$\%3=2\beta\%2+2\frac{I_e\beta\%1}{T}$$

$$\%4=e^{(\theta+\delta)(T-M)}-(\theta+\delta)(T-M)-1$$

$$\%5=e^{(\theta+\delta)T}-(\theta+\delta)T-1$$

$$\%6=e^{(\theta+\delta)T}-1-T-\frac{\delta\%5}{(\theta+\delta)}$$

$$\%7=(\alpha+C\beta)\%2+\beta(\frac{C\%6}{(\theta+\delta)T}+\frac{h\%5}{(\theta+\delta)^2T}+\frac{rC(e^{(\theta+\delta)T}-1)}{(\theta+\delta)T}+\frac{CI_c\%4}{(\theta+\delta)^2T})+\frac{I_e\alpha\%1}{T}$$

and hence $\frac{\partial NP_1(T)}{\partial T}=0$ (5.2.16)

$$\frac{\partial NP_1(T)}{\partial T}=(\frac{\%12}{\%3}-\frac{\%7\%10}{\%3^2})\%13\%2+(\frac{\%7}{\%3}-c)\%14\%2+(\frac{\%7}{\%3}-c)\%13\delta e^{(\theta+\delta)T}+\frac{A}{T^2}$$
$$-\frac{C\%14\%6}{(\theta+\delta)T}-\frac{C\%13(\%11-1-\frac{\delta(\%11-\theta-\delta)}{\theta+\delta})}{(\theta+\delta)T}+\frac{C\%13\%6}{(\theta+\delta)T^2}-\frac{h\%14\%5}{(\theta+\delta)^2T}$$
$$-\frac{h\%13(\%11-\theta-\delta)}{T(\theta+\delta)^2}+\frac{h\%13\%5}{T^2(\theta+\delta)^2}-\frac{rC\%14(e^{(\theta+\delta)T}-1)}{(\theta+\delta)T}-\frac{rC\%13(e^{(\theta+\delta)T})}{T}$$
$$+\frac{rC\%13(e^{(\theta+\delta)T}-1)}{(\theta+\delta)T^2}-\frac{CI_c\%14\%4}{(\theta+\delta)^2T}-\frac{CI_c\%13(\%8-\theta-\delta)}{(\theta+\delta)^2T}+\frac{CI_c\%13\%4}{(\theta+\delta)^2T^2}$$
$$+\frac{\%12I_e\%13\%1}{\%3T}-\frac{\%7I_e\%13\%1\%10}{\%3^2t}+\frac{\%7I_e\%14\%1}{\%3T}-\frac{\%7I_e\delta\%13\%9}{\%3(\theta+\delta)^3T}$$
$$-\frac{\%7I_e\%13\%1}{\%3T^2}$$

where

$$\%1 = 0.5M^2 - \frac{\delta(Me^{(\theta+\delta)(T-M)}(\theta+\delta)+e^{(\theta+\delta)(T-M)}-1+0.5M^2(\theta+\delta)^2)}{(\theta+\delta)^3}$$

$$\%2 = 1+\frac{\delta(e^{(\theta+\delta)T}-1)}{\theta+\delta}$$

$$\%3 = 2\beta\%2+2\frac{I_e\beta\%1}{T}$$

$$\%4 = e^{(\theta+\delta)(T-M)}-(\theta+\delta)(T-M)-1$$

$$\%5 = e^{(\theta+\delta)T}-(\theta+\delta)T-1$$

$$\%6 = e^{(\theta+\delta)T}-1-T-\frac{\delta\%5}{(\theta+\delta)}$$

$$\%7 = (\alpha+C\beta)\%2+\beta(\frac{C\%6}{(\theta+\delta)T}+\frac{h\%5}{(\theta+\delta)^2T}+\frac{rC(e^{(\theta+\delta)T}-1)}{(\theta+\delta)T}+\frac{CI_c\%4}{(\theta+\delta)^2T})+\frac{I_e\alpha\%1}{T}$$

$$\%8 = (\theta+\delta)e^{(\theta+\delta)(T-M)}$$

$$\%9 = M(\theta+\delta)^2e^{(\theta+\delta)(T-M)}+\%8$$

$$\%10 = 2\beta\delta e^{(\theta+\delta)T}-\frac{2I_e\beta\delta\%9}{(\theta+\delta)^3T}-\frac{2I_e\beta\%1}{T^2}$$

$$\%11 = (\theta+\delta)e^{(\theta+\delta)T}$$

$$\%12 = (\alpha+C\beta)\delta e^{(\theta+\delta)T}-\frac{I_e\alpha\delta\%9}{(\theta+\delta)^3T}-\frac{I_e\%1}{T^2}$$

$$+\beta[\frac{C(\%11-1-\frac{\delta(\%11-\theta-\delta)}{\theta+\delta})}{(\theta+\delta)T}-\frac{C\%6}{(\theta+\delta)T^2}+\frac{h(\%11-\theta-\delta)}{T(\theta+\delta)^2})$$

$$-\frac{h\%5}{(\theta+\delta)^2T^2}+\frac{rCe^{(\theta+\delta)T}}{T}-\frac{rC(e^{(\theta+\delta)T}-1)}{(\theta+\delta)T^2}+\frac{CI_c(\%8-\theta-\delta)}{(\theta+\delta)^2T}-\frac{CI_c\%4}{(\theta+\delta)^2T^2}]$$

$$\%13 = (\alpha-\frac{\beta\%7}{\%3})$$

$$\%14 = (-\frac{\beta\%12}{\%3}+\frac{\beta\%7\%10}{\%3^2})$$

This can be solved by Newton-Raphson's method. Call this solution to be $T_1°$. The cycle time $T = T_1°$ obtained by solving eq. (5.2.16), maximizes the net profit because

$$\frac{\partial^2 NP_1(T)}{\partial T^2}<0$$

Arguing in a similar way for case (2), we get

$$p=\frac{\%X}{\%Y}$$

where

$$\%X=(\alpha+C\beta)[1+\frac{\delta(e^{(\theta+\delta)T}-1)}{(\theta+\delta)}]+\beta[\frac{rc(e^{(\theta+\delta)T}-1)}{(\theta+\delta)T}+\frac{h}{T(\theta+\delta)^2}(e^{(\theta+\delta)T}-(\theta+\delta)T-1)$$
$$+\frac{C}{T(\theta+\delta)^2}\{(e^{(\theta+\delta)T}-1)(\theta+\delta)-T(\theta+\delta)-\delta(e^{(\theta+\delta)T}-(\theta+\delta)T-1)\}]$$
$$+\frac{\alpha I_e}{T}[\frac{\delta}{(\theta+\delta)^3}-\frac{\delta T}{(\theta+\delta)^2}-\frac{\theta T^2}{2(\theta+\delta)}+(1+\frac{\delta(e^{(\theta+\delta)T}-1)}{(\theta+\delta)})(M-T)]$$

$$\%Y=2\beta[(1+\frac{\delta(e^{(\theta+\delta)T}-1)}{(\theta+\delta)})+\frac{I_e}{T}\{\frac{\delta}{(\theta+\delta)^3}+\frac{\delta T}{(\theta+\delta)^2}+\frac{\theta T^2}{2(\theta+\delta)}+(1+\frac{\delta(e^{(\theta+\delta)T}-1)}{(\theta+\delta)})(M-T)\}]$$

and hence substituting this p in (5.2.15)

$$NP_2(T)=(-\frac{\%2}{\%1}-C)(\alpha+\frac{\beta\%2}{\%1}+\frac{\delta(\alpha+\frac{\beta\%2}{\%1})(e^{(\theta+\delta)T}-1)}{(\theta+\delta)})$$
$$-\frac{C(\alpha+\frac{\beta\%2}{\%1})[(e^{(\theta+\delta)T}-1)(\theta+\delta)-(\theta+\delta)T-\delta(e^{(\theta+\delta)T}-(\theta+\delta)T-1)]}{(\theta+\delta)}$$
$$-\frac{A}{T}-\frac{h(\alpha+\frac{\beta\%2}{\%1})(e^{(\theta+\delta)T}-(\theta+\delta)T-1)}{(\theta+\delta)^2T}-\frac{rC(\alpha+\frac{\beta\%2}{\%1})(e^{(\theta+\delta)T}-1)}{(\theta+\delta)T}$$
$$-\frac{I_e\%2(\alpha+\frac{\beta\%2}{\%1})[\frac{\delta}{(\theta+\delta)^3}-\frac{\delta T}{(\theta+\delta)^2}+\frac{\theta T^2}{2(\theta+\delta)}+(1+\frac{(e^{(\theta+\delta)T}-1)}{\theta+\delta})(M-T)]}{\%1T}$$

where

$$\%1=-2\frac{I_e\beta\delta M(e^{T(\theta+\delta)}-1)}{T(\theta+\delta)}+2\frac{\beta e^{T(\theta+\delta)}(I_e-\delta)}{\theta+\delta}+2\beta(I_e-1)-2\frac{\delta\beta(I_e-1)}{\theta+\delta}$$
$$-2\frac{I_e\delta\beta}{T(\theta+\delta)^3}+2\frac{I_e\delta\beta}{(\theta+\delta)^2}-\frac{I_e\beta T\theta}{(\theta+\delta)}-2\frac{I_e\beta M}{T}$$

$$\%2 = \alpha + \frac{(rC\beta + I_e \alpha M)e^{T(\theta+\delta)}}{T(\theta+\delta)} + \frac{\beta(h + C\theta)e^{T(\theta+\delta)}}{T(\theta+\delta)^2} + \frac{\delta(\alpha + C\beta)e^{T(\theta+\delta)}}{\theta+\delta} + C\beta$$
$$- \frac{\delta(\alpha + C\beta + h\beta - I_e \alpha(1 - \delta e^{T(\theta+\delta)}))}{\theta+\delta} - \frac{h\beta}{T(\theta+\delta)^2} - I_e \alpha - \frac{C\beta\theta}{(\theta+\delta)^2}(\frac{1}{T} + 1)$$
$$- \frac{C\beta\delta(1-\theta-\delta)}{(\theta+\delta)^2} - \frac{rC\beta + I_e \alpha\delta M}{(\theta+\delta)T} + \frac{I_e \alpha\delta}{T(\theta+\delta)^3} - \frac{I_e \alpha\delta}{(\theta+\delta)^2} + \frac{I_e \alpha T\theta}{2(\theta+\delta)}$$
$$+ \frac{I_e \alpha M}{T}$$

and again $\frac{\partial NP_2(T)}{\partial T}$ (5.2.17)

$$\frac{\partial NP_2(T)}{\partial T} = [-\frac{\%5}{\%2} + \frac{\%3\%4}{\%2^2}][\%7 + \frac{\delta(\%7)(e^{(\theta+\delta)T} - 1)}{\theta+\delta}] + (-\frac{\%3}{\%2} - C)\{\%8 + \frac{\delta\%8(e^{(\theta+\delta)T} - 1)}{\theta+\delta}$$
$$+ \delta(\%7)e^{(\theta+\delta)T}\} - \frac{C\%8((e^{(\theta+\delta)T} - 1)(\theta+\delta) - (\theta+\delta)T - \delta\%6)}{(\theta+\delta)^2 T}$$
$$- \frac{C\%7((\theta+\delta)^2 e^{(\theta+\delta)T} - \theta - \delta - \delta((\theta+\delta)e^{(\theta+\delta)T} - \theta - \delta))}{(\theta+\delta)^2 T}$$
$$+ \frac{C\%7((e^{(\theta+\delta)T} - 1)(\theta+\delta) - (\theta+\delta)T - \delta\%6)}{(\theta+\delta)^2 T^2} + \frac{A}{T^2} - \frac{h(\%8)\%6}{(\theta+\delta)^2 T}$$
$$- \frac{h\%7((\theta+\delta)e^{(\theta+\delta)T} - \theta - \delta)}{(\theta+\delta)^2 T} + \frac{h\%7\%6}{(\theta+\delta)^2 T^2} - \frac{rC\%8(e^{(\theta+\delta)T} - 1)}{(\theta+\delta)T}$$
$$- \frac{rC\%7e^{(\theta+\delta)T}}{T} + \frac{rC\%7(e^{(\theta+\delta)T} - 1)}{(\theta+\delta)T^2} - \frac{I_e\%1}{\%2T}[\%5\%7 - \frac{\%3\%7\%4}{\%2} + \%3\%8 - \frac{\%3\%7}{T}]$$
$$- \frac{I_e\%3\%7}{\%2T}[-\frac{\delta}{(\theta+\delta)^2} + \frac{\theta T}{(\theta+\delta)} + \delta e^{(\theta+\delta)T}(M - T) - 1 - \frac{\delta(e^{(\theta+\delta)T} - 1)}{(\theta+\delta)}]$$

where

$$\%1 = \frac{\delta}{(\theta+\delta)^3} + \frac{\delta T}{(\theta+\delta)^2} + \frac{\theta T^2}{2(\theta+\delta)} + \{1 + \frac{\delta(e^{(\theta+\delta)T} - 1)}{(\theta+\delta)}\}(M - T)$$

$$\%2=-2\frac{I_e\beta\delta M(e^{T(\theta+\delta)}-1)}{T(\theta+\delta)}+2\frac{\beta e^{T(\theta+\delta)}(I_e-\delta)}{\theta+\delta}+2\beta(I_e-1)-2\frac{\delta\beta(I_e-1)}{\theta+\delta}$$
$$-2\frac{I_e\delta\beta}{T(\theta+\delta)^3}+2\frac{I_e\delta\beta}{(\theta+\delta)^2}-\frac{I_e\beta T\theta}{(\theta+\delta)}-2\frac{I_e\beta M}{T}$$

$$\%3=\alpha+\frac{(rC\beta+I_e\alpha M)e^{T(\theta+\delta)}}{T(\theta+\delta)}+\frac{\beta(h+C\theta)e^{T(\theta+\delta)}}{T(\theta+\delta)^2}+\frac{\delta(\alpha+C\beta)e^{T(\theta+\delta)}}{\theta+\delta}+C\beta$$
$$-\frac{\delta(\alpha+C\beta+h\beta-I_e\alpha(1-\delta e^{T(\theta+\delta)}))}{\theta+\delta}-\frac{h\beta}{T(\theta+\delta)^2}-I_e\alpha-\frac{C\beta\theta}{(\theta+\delta)^2}(\frac{1}{T}+1)$$
$$-\frac{C\beta\delta(1-\theta-\delta)}{(\theta+\delta)^2}-\frac{rC\beta+I_e\alpha\delta M}{(\theta+\delta)T}+\frac{I_e\alpha\delta}{T(\theta+\delta)^3}-\frac{I_e\alpha\delta}{(\theta+\delta)^2}+\frac{I_e\alpha T\theta}{2(\theta+\delta)}$$
$$+\frac{I_e\alpha M}{T}$$

$$\%4=\frac{2I_e\beta}{(\theta+\delta)}[\delta e^{T(\theta+\delta)}\{\frac{M}{T^2}-\frac{M}{T}-\frac{\delta M}{T}+1+\delta-\theta-\delta M\}+\{\frac{\delta}{T^2(\theta+\delta)^2}-\frac{\theta}{2}+\frac{M(\theta+\delta)}{T^2}-\frac{\delta M}{T^2}\}]$$

$$\%5=\frac{e^{T(\theta+\delta)}}{\theta+\delta}[C\beta\{-\frac{r}{T^2}+\frac{r\theta}{T}+\frac{r}{T}-\frac{\theta}{T(\theta+\delta)}(\frac{1}{T}+\theta+1)+\delta(\theta+\delta)\}+\delta\alpha(\theta+\delta)$$
$$+h\beta\{-\frac{1}{(\theta+\delta)T^2}+\frac{\theta}{(\theta+\delta)T}+\frac{1}{(\theta+\delta)T}\}+I_e\alpha\delta\{-\frac{M}{T^2}+\frac{\theta M}{T}+\frac{\delta M}{T}-\theta-\frac{\delta}{T^2}\}]$$
$$+\frac{\beta}{(\theta+\delta)T^2}\{\frac{h}{(\theta+\delta)}+\frac{C\theta}{(\theta+\delta)}+rC\}+I_e\alpha\{-\frac{\delta}{(\theta+\delta)^3T^2}+\frac{\theta}{2(\theta+\delta)}+\frac{\delta M}{(\theta+\delta)T^2}-\frac{M}{T^2}\}$$

$$\%6=e^{(\theta+\delta)T}-(\theta+\delta)T-1$$

$$\%7=\alpha+\frac{\beta\%3}{\%2}$$

$$\%8=\beta(\frac{\%5}{\%2}-\frac{\%3\%4}{\%2^2})$$

This can be solved by Newton-Raphson's method. Call this solution to be T_2°. The cycle time $T = T_2^{\circ}$ obtained by solving eq. (5.2.17). Maximizes the net profit because

$$\frac{\partial^2 NP_2(T)}{\partial T^2}<0$$

5.3 Special Cases :

1) By putting $\theta = 0$, demand to be constant and inflation rate $r = 0$, the model reduces to that of Gupta and Vrat (1986).

2) By putting inflation rate $r = 0$ and demand to be constant R, the model reduces to that of Mandal and Phaujdar (1989).

5.4 Numerical Example and Sensitivity analysis :

Consider an inventory system with the following values of the parameters with proper units

$\alpha = 1000$, $\beta = 0.1$, $\delta = 1$, $\theta = 0.03$, $A = 250$, $r = 0.03$, $C = 20$, $h = 0.2$

$I_c = 0.15$, $I_e = 0.12$, $M = 0.0411$. (Note : $T_1^\circ = T$ in tables)

Table 5.4.1

$\alpha \backslash \beta$		0.1	0.12	0.14
1000	T	0.196	0.196	0.196
	Q	216.09	216.09	216.09
	p	50.19	41.86	35.91
	NP	33800.66	23703.99	16492.09
1200	T	0.199	0.20	0.20
	Q	263.70	265.17	265.17
	p	60.19	50.19	43.05
	NP	55541.50	41005.95	30579.05
1400	T	0.2	0.201	0.202
	Q	309.36	311.08	312.79
	p	70.19	58.53	50.19
	NP	82109.96	62309.39	48157.26

Table 5.4.2

$\alpha\setminus\delta$		1.0	1.2	1.4
1000	T	0.196	0.139	0.11
	Q	216.09	150.83	118.52
	p	50.19	50.23	50.26
	NP	33800.66	29246.19	26154.81
1200	T	0.199	0.143	0.113
	Q	263.70	186.67	146.43
	p	60.19	60.23	60.26
	NP	55541.50	49887.14	46069.07
1400	T	0.2	0.145	0.114
	Q	309.36	221.11	172.48
	p	70.19	70.23	70.26
	NP	82109.96	75263.15	70580.61

Table 5.4.3

$\beta\setminus\delta$		1.0	1.2	1.4
0.1	T	0.196	0.139	0.11
	Q	216.09	150.83	118.52
	p	50.19	50.23	50.26
	NP	33800.66	29246.19	26154.81
0.12	T	0.196	0.139	0.112
	Q	216.09	150.83	120.86
	p	41.86	41.89	41.92
	NP	23703.99	19440.29	16579.47
0.14	T	0.196	0.139	0.113
	Q	216.09	150.83	122.02
	p	35.91	35.94	35.97
	NP	16492.09	12436.11	9687.65

Table 5.4.4

$\alpha\setminus\theta$		0.03	0.04	0.05
1000	T	0.196	0.193	0.189
	Q	216.09	212.66	208.00
	p	50.19	50.20	50.20
	NP	33800.66	33478.87	33119.54
1200	T	0.199	0.195	0.192
	Q	263.70	258.12	253.99
	p	60.19	60.20	60.20
	NP	55541.50	55055.98	54636.84
1400	T	0.2	0.197	0.193
	Q	309.36	304.55	298.02
	p	70.19	70.20	70.20
	NP	82109.96	81571.02	80948.82

Table 5.4.5

$\beta \backslash \theta$		0.03	0.04	0.05
0.1	T	0.196	0.193	0.189
	Q	216.09	212.66	208.00
	p	50.19	50.20	50.20
	NP	33800.66	33478.77	33119.54
0.12	T	0.196	0.192	0.189
	Q	216.09	211.44	208.00
	p	41.86	41.86	41.86
	NP	23703.99	23379.99	23090.1
0.14	T	0.196	0.192	0.189
	Q	216.09	211.44	208.01
	p	35.91	35.91	35.91
	NP	16492.09	16195.71	15926.24

Table 5.4.6

$\delta \backslash \theta$		0.03	0.04	0.05
1.0	T	0.196	0.193	0.189
	Q	216.09	212.66	208.01
	p	50.19	50.20	50.20
	NP	33800.66	33478.77	33119.54
1.2	T	0.139	0.138	0.136
	Q	150.83	149.75	147.50
	p	50.23	50.23	50.23
	NP	29246.19	29052.15	28806.82
1.4	T	0.11	0.109	0.109
	Q	118.52	117.43	117.49
	p	50.26	50.26	50.26
	NP	26154.81	25982.42	25878.65

Table 5.4.7

$\alpha \backslash r$		0.03	0.04	0.05
1000	T	0.196	0.197	0.197
	Q	216.09	217.31	217.31
	p	50.19	50.20	50.20
	NP	33800.66	33620.75	33400.13
1200	T	0.199	0.199	0.20
	Q	263.70	263.71	265.17
	p	60.19	60.19	60.20
	NP	55541.50	55276.51	55073.32
1400	T	0.2	0.2	0.201
	Q	309.36	309.37	311.08
	p	70.19	70.20	70.20
	NP	82109.96	81800.68	81579.41

Table 5.4.8

$\beta \setminus r$		0.03	0.04	0.05
0.1	T	0.196	0.197	0.197
	Q	216.09	217.31	217.31
	p	50.19	50.20	50.20
	NP	33703.99	33620.75	33400.13
0.12	T	0.196	0.197	0.197
	Q	216.09	217.31	217.31
	p	41.86	41.86	41.86
	NP	23703.99	23513.94	23293.32
0.14	T	0.196	0.197	0.198
	Q	216.09	217.31	218.53
	p	35.91	35.91	35.91
	NP	16492.09	16294.80	16097.23

Table 5.4.9

$\delta \setminus r$		0.03	0.04	0.05
1.0	T	0.196	0.197	0.197
	Q	216.09	217.31	217.31
	p	50.19	50.20	50.20
	NP	33800.66	33620.75	33400.13
1.2	T	0.139	0.14	0.141
	Q	150.83	152.01	153.19
	p	50.23	50.23	50.23
	NP	29246.19	29082.06	28917.52
1.4	T	0.11	0.111	0.112
	Q	118.52	119.69	120.86
	p	50.26	50.26	50.26
	NP	26154.81	26006.15	25856.88

Table 5.4.10

$r \setminus \theta$		0.03	0.04	0.05
0.03	T	0.196	0.193	0.189
	Q	216.09	212.66	208.00
	p	50.19	50.20	50.20
	NP	33800.66	33478.87	33119.54
0.04	T	0.197	0.193	0.19
	Q	217.31	212.66	209.22
	p	50.20	50.20	50.20
	NP	33620.75	33258.4	32940.13
0.05	T	0.197	0.194	0.19
	Q	217.31	213.88	209.22
	p	50.20	50.20	50.20
	NP	33400.13	33078.53	32719.9

Table 5.4.11

$\alpha\backslash M$		15/365	30/365	45/365
1000	T	0.196	0.197	0.199
	Q	216.09	217.31	219.75
	p	50.19	50.20	50.20
	NP	33800.66	34008.20	34277.55
1200	T	0.199	0.2	0.202
	Q	263.70	265.17	268.11
	p	60.19	60.20	60.20
	NP	55541.50	55821.50	56200.15
1400	T	0.2	0.201	0.203
	Q	309.36	311.08	314.51
	p	70.19	70.20	70.20
	NP	82109.96	82473.46	82981.34

Table 5.4.12

$\beta\backslash M$		15/365	30/365	45/365
0.1	T	0.196	0.197	0.199
	Q	216.09	217.31	219.75
	p	50.19	50.20	50.20
	NP	33800.66	34008.20	34277.55
0.12	T	0.196	0.197	0.199
	Q	216.09	217.31	219.75
	p	41.86	41.86	41.86
	NP	23703.99	23888.59	24116.73
0.14	T	0.196	0.197	0.199
	Q	216.09	217.31	219.75
	p	35.91	35.91	35.91
	NP	16492.09	16660.32	16859.02

Table 5.4.13

$\delta\backslash M$		15/365	30/365	45/365
1.0	T	0.196	0.197	0.199
	Q	216.09	217.31	219.75
	p	50.19	50.20	50.20
	NP	33800.66	34008.20	34277.55
1.2	T	0.139	0.141	0.144
	Q	150.83	153.19	156.75
	p	50.23	50.23	50.23
	NP	29246.19	29531.63	29899.12
1.4	T	0.11	0.112	0.116
	Q	118.52	120.86	125.54
	p	50.26	50.25	50.25
	NP	26154.81	26480.60	26973.54

Table 5.4.14

$\theta \backslash M$		15/365	30/365	45/365
0.03	T	0.196	0.197	0.199
	Q	216.09	217.31	219.75
	p	50.19	50.19	50.20
	NP	33800.66	34008.20	34277.55
0.04	T	0.193	0.194	0.196
	Q	212.66	213.80	216.31
	p	50.20	50.20	50.20
	NP	33478.87	33666.81	33957.11
0.05	T	0.189	0.19	0.192
	Q	208.00	209.22	211.65
	p	50.20	50.20	50.20
	NP	33119.54	33328.33	33599.92

Observations:

The effects of various parameters on variables and objective function can be interpreted from the above tables in the following tabular form.

Note :- ↑ : Increase, ↓ : Decrease, ⇑ : Significant Increase, ⇓ : Significant Decrease

↑ ~ : Marginal Increase, ↓ ~ : Marginal Decrease, -- : No Change

Keeping Constant		β	θ	r	M	δ
Increasing α	T	↑	↑	↑	↑	↑
	Q	↑	↑	↑	↑	↑
	P	↑	↑	↑	↑	↑
	NP	⇑	⇑	⇑	⇑	⇑

Keeping Constant		θ	r	M	δ
Increasing β	T	--	--	--	--
	Q	--	--	--	--
	p	↓	↓	↓	↓
	NP	⇓	↓	⇓	⇓

Keeping Constant		θ	r	M
Increasing δ	T	↓	↓	↓
	Q	↓	↓	↓
	p	↓	↑	↑ ~
	NP	↓	↓	↓

Keeping Constant		θ
Increasing r	T	↑ ~
	Q	↑ ~
	p	↑ ~
	NP	↓ ~

Keeping Constant		M
Increasing θ	T	↓ ~
	Q	↓
	p	↑ ~
	NP	↓

5.5 Conclusions :

In this chapter, a mathematical model is developed with stock-dependent and selling price dependent demand when supplier offers permissible delay in payments. It is observed that the results of sensitivity analysis are very consistent with the prevailing economic incentives. Increase in the delay period results in significant increase in the net profit of the customer. Increase in deterioration rate of units in the inventory system decreases the net profit, keeping δ or θ under control, elongated delay period may result in increase in the net profit. In future research, this problem can be extended to time dependent deterioration rate.

Directions for future research

- The models proposed in chapter 3 can be extended by taking demand to be a function of selling price, time varying and stock dependent. Also they can be generalized to allow for shortages and inflation rates.
- The model proposed in chapter 4 can be extended to a two parameter Weibull distribution deterioration. Further it can be extended by allowing shortages, quantity discounts and inflation rates.
- The model in chapter 5 can be extended to time dependent deterioration rate.

LIST OF PAPERS PUBLISHED, ACCEPTED, PRESENTED AND SUBMITTED

- **List of papers published:**

1. *An order level lot-size model with time dependent deterioration and permissible delay in payments*, Advances and Applications in Statistics, 3(2), 2003, 159-172.

- **List of papers accepted for publication:**

1. *A lot-size model for items with time dependent deterioration*, accepted by Industrial Engineering Journal, Bombay.
2. *An order level lot-size model with time dependent deterioration*, accepted by Current Sciences, Dumka.

- **List of papers presented:**

1. *An order level lot-size model with time dependent deterioration*, presented at the 2nd Gujarat Research Students Meet on 26th February 2003 at Vallabh Vidyanagar. **(GSRSM-2)**
2. *An order level lot-size model with time dependent deterioration and permissible delay in payments*, presented at the 6th Annual conference of the Society of Operations Management held on 20th December 2002 at the Indian Institute of Management, Kozhikode. **(SOM-6)**
3. *An EOQ Model for deteriorating items with two parameter weibull distribution under supplier credits,* presented at the 24th Annual Conference of the Gujarat Statistical Association held on 11th October 2003 at Department of Statistics, Bhavnagar. **(GSA-24)**
4. *A lot-size Model with variable deterioration rate under supplier credits,* submitted and accepted in the International Conference of the Asia-Pacific Operational Research Society to be held at New Delhi during 8-11th December 2003. **(APORS-2003)**
5. *An EOQ model for deteriorating items with selling price and stock dependent demand during inflation under supplier credits*, accepted for presentation in the 7th Annual Conference of the Society of Operations Management to be held at Indian Institute of Management, Indore during 19-21st December, 2003. **(SOM-7)**
6. *An EOQ Model for deteriorating items with price dependent demand and permissible delay in payments under inflation,* accepted for presentation at the International Conference on Operations research and Economics, hosted by Indian Statistical Institute (ISI) Calcutta, during 8-10th January, 2004. **(ICOR-2004)**

- **List of papers submitted:**

1. *An EOQ Model for deteriorating items with price dependent demand under supplier credits,* submitted to International Journal of Systems Science, UK.
2. *An EOQ Model for deteriorating items with two parameter weibull distribution under supplier credits,* submitted to Asian Journal of Operations Management, India.
3 *An EOQ model for deteriorating items with selling price and stock dependent demand during inflation under supplier credits,* submitted to Opsearch, India.
4. *A lot-size Model with variable deterioration rate under supplier credits,* submitted and accepted in the International Conference of the Asia-Pacific Operational Research Society to be held at New Delhi in December 2003.

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